

## **ANALYSIS OF ACCURACY AND FORMULATION OF CRITERION FOR COMPARING TWO VARIANTS OF TRILATERATION METHOD OF ACOUSTIC LOCATION**

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**Abstract:** In the article the question on development the criterion for comparison of two variants of realization of trilateration method of acoustic location has been considered. It is shown, that upon fulfillment of some condition and sufficiently low level of noises in the signal the choice of the signal strength measuring techniques of trilateration method of location can be preferable.

**Key words:** acoustic location; accuracy; criterion; technological explosions; extinction; sound speed; model; trilateration; sound strength measurements

### **Introduction**

According to work [1] location technology is a combination of methods and techniques for determining a physical location of an object in the real world.

Six fundamental techniques are known as suitable for location of wave generating objects: proximity, trilateration, hyperbolic lateration, triangulation, fingerprinting, and dead reckoning.

The method of trilateration makes it possible to compute the position of object at the basis of results of measurements of distances between reference points with known positions and the object. In order to estimate the distance between them, it is necessary to measure the passing time of the signal or to measure the strength of the signal at the reference points.

In triangulation method the angle of arrival of signals coming from the object to receivers are measured to locate the object. Two angles measured at two reference points make it possible to form two lines to define the objects location at the point of intersection.

The most popular and used in acoustic location are triangulation and trilateration methods. In this paper we shall compare and analyze the accuracy of basic variants of trilateration method and suggest the new criterion for choosing any of them for development of three points system of acoustic location. In next sector we shall review the most typical realizations of location using trilateration methods.

### **Review of cases of typical utilization of two techniques of trilateration method in location systems**

There are some practical tasks, solution of which is formalized as an independent direction of location procedure. For example in military operations, sound ranging is used for determining the location of the enemy artillery using results of processing of received sound waves of its guns firing.

According to work [2], a new Offshore Test and Training Area (OTTA) was initiated for testing, training and evaluating standoff weapons. One subsystem deployed in the OTTA was a GPS and acoustical buoy system capable of determining the impact time and location of a weapon within the marine test range.

Acoustic event data collected by the buoys is relayed to a command and control station that uses a trilateration algorithm to compute a target score for the event. Buoy real-time scoring is provided an accuracy of 7 meters while post-processed scores are accurate to 3 meters. Two weapon tests were conducted in 2004 using the GPS acoustic buoy system. In both cases the impact scores determined from the system were within 1 standard deviation of GPS-derived truth impact scores.

As it was mentioned above, using the measuring of signal strength is widely used in solution some location problems by way of trilateration method. In the work [3] the method of local signal strength gradient is described. In this realization of trilateration method the distances from the signal source to measurement points are estimated and combined to infer the location of object. The path losses from transmitter to receiver are defined as a function of distance. The gradient calculation is based on the concept that the signal strength indirectly reflects the coming signals direction.

In another task of location of wave generating object in indoor environment a weighted center of mass based trilateration approach was used [4]. The suggested approach consists of two phases: (1) determination of distance using signal strength of all access points as received by the mobile device, (2) Determination the most probable location of object using coordinates of access points and calculated distances from the object to those points. The suggested algorithm guarantees location of mobile object in indoor environment where the configuration of access points is not fixed and the movements in signal attenuating environment is unpredictable.

Relatively simple realization of trilateration method is suggested in the work [5], which presents the design of localization system for a team of centimeter-scale robots that collaborate to map and explore unknown environments. The localization system uses ultrasound to measure the distance from each moving robot to three stationary robots that serve as beacons. From these distance measurements the position of the robots is derived using a trilateration algorithm. The robot team can move over large distance by using a leap-frogging approach in which different robots serve as beacons at different times. The localization system is able to obtain position estimates more accurate than can achieved through dead reckoning.

As it is noted in work [6], the multilateration, as a basic building block of localization, however, has not yet overcome the challenges of 1) poor ranging measurements; 2) dynamic and noisy environments, and 3) fluctuations in wireless communications. Hence, multilateration-based approaches often suffer from poor accuracy and can hardly be employed in practical applications. Authors propose Quality of Trilateration (QoT) parameter that quantifies the geometric relationship of objects and ranging noises. This work suggests a confidence-based iterative localization scheme, in which nodes dynamically select trilaterations with the highest quality for location computation.

As it is noted in the work [7] in 2D trilateration, the locations of three base points are known as well as the distances from each of these base points to the object to be localized. Looked at visually 2D trilateration involves finding the location where three circles intersect. Thus, to locate a robot using 2D trilateration the sensing robot must know the locations of three points in its own coordinate system and be able to measure distances from these three points to the sensed robot.

Thus, as it can be seen from the composed review of recently developed location systems, various modifications of basic two techniques of trilateration method are widely used for solution of localization tasks.

### Problem formulation

It is well – known, that the coordinates of sources of millisecond scale infrasound waves of pulse type occurring upon technological explosions may be found using classic trilateration method, based on at least three microphones [8].

It is known [9], that propagation and extinction of plane acoustic wave's energy may be formulated with sufficient accuracy as follows

$$F_1 = F_0 e^{-\frac{mx}{2}}, \quad (1)$$

where  $F$  - pressure of acoustic signal at distance  $x$  apart from source of signal;  $F_0$  - pressure of acoustic signal at the source of signal;  $m$  - parameter, characterizing the extinction of the signal.

On the basis of formula (1) one can suggest the alternative techniques for realization of trilateration method of location, where the basic parameter for all necessary measurements is not speed of sound, but parameter featuring of extinction of sound's energy.

To explain the similarity two possible realizations of trilateration methods we use the generalized model of three-points acoustic location (figure 1). The first variant of trilateration method provides for measuring distances between the object and microphones. The second variant suggests measuring of signal strength parameter and may be realized by way of iterative calculation of  $R_i (i=1,3)$  using the initial presumptions about value  $F_0$ . To calculate  $R_i (i=1,3)$  we use the formula (1), where as a  $F_i$  we use the measured values of the sound pressure at relevant distances, and the parameter  $x$  is substituted with  $R_i$ . Initially, upon predicted value of  $F_{0i}$  we calculate values  $R'_1, R'_2$  and  $R'_3$  (figure 1). Then we draw circles centers of which coincides with  $M_1, M_2$  and  $M_3$  and step-by-step increase value the  $F_{0i}$  as far as the contours of three circles would contact each – other at the single point. This point should be considered as a target object site.

The problem is that one of two variants for realization of trilateration method is be selected on the basis of reliable criterion which should be developed.

### The problem solution

Obviously that to choose one of these two techniques, the analysis of accuracy of them should be done. It should be stressed out, that the major factors effecting on accuracy of functioning of acoustic location systems are temperature and humidity. At the same time, the analysis held in [9] has shown, that the effect of temperature is more strong than effect of humidity (at least 3 times) in the allowable intervals of variation of these parameters.

Due to this reason further we shall consider the effect of temperature on the accuracy of location.

Now we shall evaluate the effect of temperature on accuracy of location via effects on sound's speed and parameter of extinction in two techniques.

The error of location  $\Delta_1 x$  due to increase of sound's speed under effect of temperature may be evaluated as

$$\Delta_1 x = T_0 \cdot \Delta v. \quad (2)$$

where  $\Delta v$ -increment of sounds speed under effect of temperature;  $T_0$  – registered sound wave propagation time.

Then we consider the error of location in the trilateration method where the parameter  $m$  increases as far as  $m_1 = m + \Delta m$  due to temperature effect. In this case

$$F_2 = F_0 e^{-\frac{(m+\Delta m)x}{2}}. \quad (3)$$

Where  $F_2$ .- pressure of acoustic signal at point  $x$  upon increased temperature.

Logarithm of equation (3) gives us

$$\ln\left(\frac{F_0}{F_2}\right) = \frac{x \cdot m}{2} + \frac{x \cdot \Delta m}{2}. \quad (4)$$

Logarithm of equation (1) gives us

$$\ln\left(\frac{F_0}{F_1}\right) = \frac{x \cdot m}{2}. \quad (5)$$

Assume that the condition  $F_1 = F_2$  is met. This does mean that  $x$  is changed as far as  $x - \Delta_2 x$ .

Therefore, the equation (4) can be written as

$$\ln\left(\frac{F_0}{F_2}\right) = \frac{x \cdot m}{2} + \frac{x \cdot \Delta m}{2} - \Delta_2 x \left(\frac{m}{2} + \frac{\Delta m}{2}\right). \quad (6)$$

From equation (6) we get

$$\Delta_2 x = x - \frac{\ln\left(\frac{F_0}{F_2}\right)}{\frac{m}{2} + \frac{\Delta m}{2}} \quad (7)$$

Taking into consideration equations (2) and (7) and forming the condition

$$\Delta_1 x > \Delta_2 x, \quad (8)$$

we get

$$\frac{\Delta v}{v} > \frac{\Delta m}{m + \Delta m}. \quad (9)$$

It should be noted, that the initial condition (8) is valid for prevalence of signal strength measurements technique for realization of acoustic location. The resulting formula (9) is inequality, characterizing the suggested condition for prevalence of signal strength measurement technique upon realization of trilateration method of location. It should be noted, that the condition (9) is obtained on the basis of comparing distance measuring errors, i.e. inequality (8). The analogical condition can be obtained on the basis of signal/noise (SNR) assessment. Taking into account the formulas (1) and (2) the real signal  $F_{1s}$  in distance measuring variant of trilateration method may be calculated as

$$F_{1s} = F_0 e^{\frac{-m(T_0 \cdot v + T_0 \Delta v)}{2}} \quad (10)$$

The noise signal  $F_{1n}$  may be calculated as

$$F_{1,n} = F_0 e^{\frac{-mx}{2}} - F_0 e^{\frac{-m(x+T_0 \Delta v)}{2}} \quad (10)$$

where  $x = T_0 \cdot v$ .

Taking into account the formula  $e^{-x} \approx 1 - x$  upon  $x < 0,1$  from the formulas (9) and (10) we get

$$SNR_1 = \frac{F_0 e^{\frac{-mx}{2}}}{F_{1,n}} \approx \frac{2}{m T_0 \Delta v} \quad (11)$$

The similar formula can be obtained for signal strength measuring variant of trilateration method. In line with formulas (1) and (3) the noise signal may be estimated as

$$F_{2,n} = F_0 e^{\frac{-mx}{2}} (1 - e^{\frac{-\Delta mx}{2}}) \quad (12)$$

Signal/noise ratio may be calculated as

$$SNR_2 = \frac{1}{1 - e^{-\frac{\Delta m x}{2}}} \approx \frac{2}{\Delta m \cdot x} \quad (12)$$

Taking into consideration the formulas (11) and (12) we can get another condition for prevalence of signal strength measuring technique of trilateration method:

$$\frac{m}{\Delta m} > \frac{x}{\Delta x} \quad (13)$$

Comparing the formulas (9) and (13) one can decide that the formula (13) is more strict condition for choosing the signal strength method.

### Experimental and model researches

Thus, the formulas (9) and (13) are the suggested criteria for selection signal strength measurements technique for location of objects in condition of non-noisy reception. But in reality, the noisiness of receipt signal demands the strict stratification of signal on authenticity. Total noises existing in measuring signal also includes the noises in quantification carried out by analog-digit convertor.

As regard the signal strength measurements in trilateration method of location it should be noted, that the authenticity of quantification and order of coder finally will restrict the geometric accuracy of the location system, and upon 12 bites order and authenticity equal to 0,5 the achievable geometric accuracy at the distance 10 km will be equal to  $\pm 2$  m.

It is obvious that for further increase of accuracy the signal/noise ratio and the order of analog-digital converter should be increased.

The experimental researches were held on following methodology. Acoustic pulse generator and the microphone were installed at the test field at distance 600 m from each-other. The average value of measured time of passage of acoustic pulse through the distance was equal to 1,92 sec.

Experiments were held 3 times during the day of 9<sup>00</sup>, 10<sup>30</sup> and 12<sup>00</sup> hours a.m. The measured average temperature during experiments were 15°; 20° and 24° C. The relative humidity changed between 85% - 80%, but was considered as constant. The values of sound speeds increments taken from [9] are shown in the table 1. The values of  $\Delta_1 x$  calculated using formula (2) are also shown in the table 1.

Table 1. Values of major parameters of experimental research

$T\text{ }^{\circ}\text{C}$	15 °C	20 °C	24 °C
$\Delta v$	0,165%	0,3%	0,42%
$\Delta_1 x$	$1,92 \times 0,165\% \times 331 \frac{m}{sec} = 0,89m$	$1,92 \times 0,3\% \times 331 \frac{m}{sec} = 1,61m$	$1,92 \times 0,42\% \times 331 \frac{m}{sec} = 2,35m$
$\Delta_2 x$	1,20 m	1,72 m	2,13
$N_i$	500	342	281

Values of  $\Delta_2 x$  were determined as follows.

Amplitudes of sent acoustic signals by frequency of 3 kHz were measured at the point  $x = 600$  m, after the time interval  $T = 1,92$  sec passed the pulse discharge. The value of standard deviation of noises  $\sigma$  was equal to  $0,5 \pm 0,1$  mV. Then we calculate the number  $N_i$  of separable values of received signal using formula

$$N_i = \frac{U_i}{\Delta U}; i = \overline{1,3},$$

where  $U_i$ - is amplitude of received signal at distance  $x = 600$  m within selected day time intervals;  $\Delta U$ - value of least quant of used analog-digital converter;  $\Delta U = 2\sigma = 1mB$ .

The value of  $\Delta_2 x$  were calculated as

$$\Delta_2 x = \frac{600m}{N_i}; i = \overline{1,3}.$$

The calculated values of  $\Delta_2 x$  and  $N_i$  are also included in table 1. The graphics of  $\Delta_1 x$  and  $\Delta_2 x$  depending on T are shown on figure 2. As it can be seen from figure 2 by increase of temperature relative decrease of  $\Delta_2 x$  was fixed which may be considered as confirmation of presence of possibility to select any of two considered location method depending on air temperature.



### **Discussions and conclusion**

As it was noted in this paper the trilateration method of acoustic location may be implemented using two techniques: (1) by measuring strength of received signal, and (2) by measuring of radius-distances from the object to microphones.

It is shown, that the similarity of error analysis of two realization of trilateration methods does exist. At the same time the principal differences exist between these variants of realization of trilateration method. In this paper we formulate two simple criteria for choosing the most preferable variant in regard of accuracy of acoustic location. An experimental confirmation of the formulated criteria is carried out using non-direct model and experimental researches to show the possibility of choosing any one of two acoustic location techniques with minimum error of location.. As a conclusion it should be noted that existence of two different comparable techniques of realization of trilateration method is powerful incentive to develop the combined location system using alternatively both techniques of location depending on environmental conditions.

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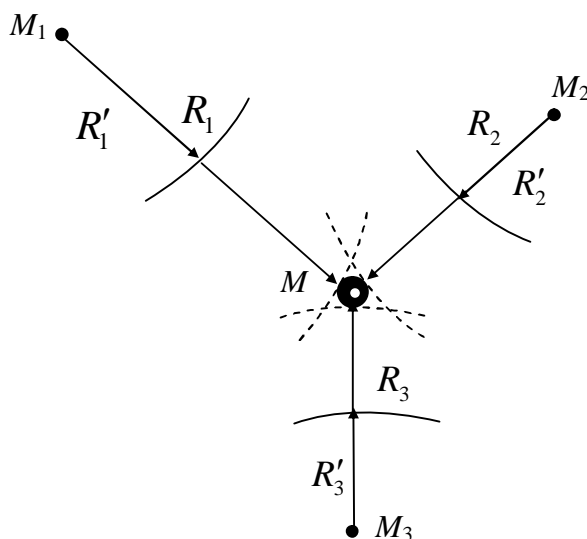


Figure 1. The generalized model of acoustic location.

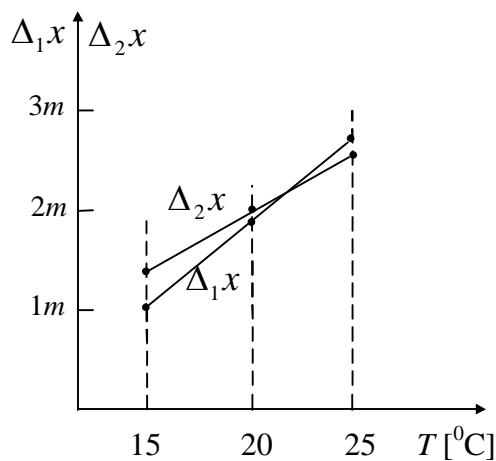


Figure 2. Curves of dependences of  $\Delta_1 x$  and  $\Delta_2 x$  on temperature.