Using Analogy in Teaching Mathematics: An Investigation of Mathematics Education Students in School of Education - Can Tho University

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Abstract

Mathematics education students in School of Education - Can Tho University (Vietnam) have had opportunities to learn how to use analogical reasoning in teaching mathematics. The question is: "Do the students prefer to select analogy as strategy for teaching mathematics?" The results of the investigation of 52 mathematics education students showed that the students paid little attention to applying analogy to teaching mathematics.

Key words: Teaching with analogy, analogical reasoning, coordinate method, active learning, mathematics education, TWA model.

1. Introduction

In School of Education - Can Tho University (Viet Nam), analogical reasoning and the important role of analogy in teaching mathematics have been mentioned in many courses in mathematics teacher training program of four years. The course "Lý luận dạy học toán học" (Didactics of Mathematics Instruction) (Loc, 2003) introduced analogy as inductive reasoning; the course " Hoạt động dạy và học môn Toán" (Teaching Activities and Learning Activities in Mathematics) (Loc, 2008) showed the relationships between the use of analogy in mathematics learning and theory of constructivism; the course "Xu hướng dạy học không truyền thống" (Non -traditional trends in instruction) (Loc, 2007) introduced model TWA (Teaching with analogy) for using analogies in teaching mathematics. In order to know how actual ability of mathematics education students in applying analogical reasoning to teaching mathematics is, we conducted a study with two research questions as follows:

Research questions:

1. While work personally, do mathematics education students have priority to use analogy for teaching some contents in the topic "Coordinate method in space"?

2. Through working in group, do the students increase the use of analogy in teaching some contents in the topic "Coordinate method in space"?

2. Background

According to Wikipedia, "analogy is a cognitive process of transferring information or meaning from a particular subject (the analog or source) to another particular subject (the target), or a linguistic expression corresponding to such a process". Analogy plays important role in theory of learning. Many educators argued that analogical reasoning becomes an effective learning - teaching tool. Brown (2003) argued that "analogy as a learning mechanism is a crucial factor in knowledge acquisition at all ages" (p.370). Halford (1993) considered analogy as the core of cognitive development in his structure – mapping theory. Cognitive development, according to Halford, consists of assigning elements of one structure to elements of another structure by corresponding relationships. Loc (2008) said that analogy is a tool helping students constructing knowledge thanks to processes of formulating and testing hypothesis, and analogy is a tool assisting teachers to predict errors of students in teaching mathematics (Loc, 2007).

About how to use analogy in science teaching, the mathematics education students in our school have learned the model TWA (Teaching with analogy), (Glynn, 2007).The model consists of six steps as follows:

Step 1: Introduce the target concept,

Step 2: Review the analog concept,

Step 3: Identify relevant features of the target and analog,

Step 4: Map similarities,

Step 5: Indicate where the analogy breaks down, and

Step 6: Draw conclusions.

In our opinion, the model is not difficult for teachers to apply to teaching mathematics in high school.

3. Coordinate method in mathematics curriculum for high schools in Vietnam

In Vietnam, high school students learn some topics on Coordinate method of Analytic Geometry in Grade 10 and Grade 12. In Grade 10, topics on plane analytic geometry are written in textbook "Hình Học 10" (Geometry 10); in Grade 12, topics on Coordinate

method in space occur in textbook "Hình Học 12" (Geometry 12). As we know, many of the content knowledge on coordinate method in space is analogical to the one in plane according to structural aspect (see Table 1). Therefore, teaching the topics on coordinate method in Geometry 12 with analogy could be an effective teaching strategy which mathematics teachers often use in teaching Geometry 12.

Table 1: Analogies between Coordinate Axis in Plane and in Space

Content knowledge	**Coordinate Axis in Space – Geometry 12 textbook** (*Target knowledge*)	**Coordinate Axis in Plane – Geometry 10 textbook** (*Analog*)
0. Beginning the lesson	Introducing overview of Coordinate Method in Space	Coordinate Method in Plane
1. Coordinate axis	Coordinate Oxyz $(O,\vec{i},\vec{j},\vec{k})$	Coordinate Oxyz $(O,\vec{i},\vec{j})$
2. Coordinates of vectors	$\vec{u}=(x;y;z)\Leftrightarrow \vec{u}=x.\vec{i}+y\vec{j}+z\vec{k}$ Given: $\vec{u}=(x;y;z),\vec{v}=(x',y',z')$ $\vec{u}=\vec{v}\Leftrightarrow\begin{cases}x=x'\\y=y'\\z=z'\end{cases}$ $\vec{u}+\vec{v}=(x+x';y+y';z+z')$ $\vec{u}-\vec{v}=(x-x';y-y';z-z')$ $k\vec{u}=(kx;ky;kz)$ $\vec{u}.\vec{v}=x.x'+y.y'+z.z'$ $\vec{u}\perp\vec{v}\Leftrightarrow x.x'+y.y'+z.z'=0$ $\cos(\vec{u},\vec{v})=\frac{xx'+yy'+zz'}{\sqrt{x^2+y^2+z^2}.\sqrt{x'^2+y'^2+}}$	$\vec{u}=(x;y)\Leftrightarrow \vec{u}=x.\vec{i}+y\vec{j}$ Given: $\vec{u}=(x;y),\vec{v}=(x',y')$ $\vec{u}=\vec{v}\Leftrightarrow\begin{cases}x=x'\\y=y'\end{cases}$ $\vec{u}+\vec{v}=(x+x';y+y')$ $\vec{u}-\vec{v}=(x-x';y-y')$ $k\vec{u}=(kx;ky)$ $\vec{u}.\vec{v}=x.x'+y.y'$ $\vec{u}\perp\vec{v}\Leftrightarrow x.x'+y.y'=0$ $\cos(\vec{u},\vec{v})=\frac{xx'+yy'}{\sqrt{x^2+y^2}.\sqrt{x'^2+y'^2}}$
3. Coordinates of a point	$M(x;y;z)\Leftrightarrow \overrightarrow{OM}=x.\vec{i}+y\vec{j}+z\vec{k}$ Given : $M(x;y;z),N(x';y';z')$ $\overrightarrow{MN}=(x'-x;y'-y;z'-z)$ $MN=\sqrt{(x'-x)^2+(y'-y)^2+(z'-z)^2}$	$M(x;y)\Leftrightarrow \overrightarrow{OM}=x.\vec{i}+y\vec{j}$ Given : $M(x;y),N(x';y')$ $\overrightarrow{MN}=(x'-x;y'-y)$ $MN=\sqrt{(x'-x)^2+(y'-y)^2}$
4. The equation of a sphere	The equation of a sphere: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2$	The equation of a circle $(x-x_0)^2+(y-y_0)^2=R^2$,

	$x^2+y^2+z^2-2ax-2by-2cz+d=0$ Center I(a,b,c), Radius $R=\sqrt{a^2+b^2+c^2-d}$	$x^2+y^2-2ax-2by+c=0$ Center I(a,b), Radius $R=\sqrt{a^2+b^2-c}$

4. Research design

Subjects: 52 mathematics education students of the School of education, Can Tho University, Vietnam. These students are studying in final year of university level.

Tasks for students:

Stage one (work personally): Each student studies textbooks Geometry 12, afterward compose lesson plans to teach some of content knowledge in "Coordinate Method in Space" which we selected for our investigation: 1.Coordinate Axis in Space; 2. Coordinates of a vector ; 3. Coordinates of a point; 4.The equation of a sphere (see Table 1)

Stage two (working in group): Students cooperate to compose lesson plans to teach the same content knowledge in "Coordinate Method in Space" as the stage one.

Rubric for evaluating lesson plan

The rubric for evaluating lesson plan was created on basis of TWA model (Table 2).

Table 2: Rubric for evaluating lesson plan

Mark	Using analogy into teaching
0	Not using any anlogogy.
1	Only talking about analog
2	Recalling characteristics of analog, but not establishing any correspondence between analog and target
3	Establishing correspondences between analog and target
4	Drawing a conclusion about the analogy and comparison of the new material with the already learned material

5. Results and discussion

***For the research question 1*:**

The Table 3 shows the marks which the students had after composing lesson plan by personally.

Comment: If let a = (0+1+2+3+4)/5 (=2), all of mean values in Table 3 are under a. Therefore, strategy "The use of analogy for teaching" was not a preference of the students.

Table 3: The level of using analogy in teaching of mathematics education students

Content knowledge	**Using analogies in teaching**					**Mean**
	0	**1**	**2**	**3**	**4**	
0.Beginning the lesson (S_0)	42 (students)	10	0	0	0	**0.19**
1.Coordinate Axis in Space (S_1)	6	23	19	4	0	**1.40**
2. Coordinates of a vector (S_2)	23	10	15	4	0	**1.00**
3. Coordinates of a point (S_3)	35	4	11	2	0	**0.62**
4.The equation of a sphere (S_4)	22	9	17	4	0	**1.06**

***For the research question 2*:**

The Table 4 presents the levels of using analogy while working personally and in group.

Table 4: The level of using analogy in teaching of mathematics education students while working personally and in group.

Group		**Group Mean in terms of working personally**					**Group Mean in terms of group discussion**				
Group	Students	S_0	S_1	S_2	S_3	S_4	S_0	S_1	S_2	S_3	S_4
1	Sơn	1	2	2	0	2	1	2	3	2	0
	Bằng	0	0	0	0	0					
	Tú	0	1	3	3	3					
	Mean	**X_1 = 1.13**					**Y_1 = 1.60**				
2	Tiến	1	2	0	2	3	1	1	0	2	0
	M.Liền	1	1	0	0	2					
	Hiền	1	1	2	0	0					
	Mean	**X_2 = 1.70**					**Y_2 = 0.90**				
3	L.Trâm	0	3	1	2	0	0	3	2	0	2
	Trang	0	3	2	0	2					
	Long	0	3	2	2	2					
	Mean	**X_3 = 1.47**					**Y_3 = 1.40**				
4	Xuân	0	2	2	2	3	0	2	2	2	3

	Hào	0	1	0	1	0					
	L.Em	0	0	0	0	0					
	Mean	**X_4= 0.73**					**Y_4 = 1.80**				
5	Chăm	0	1	0	0	0	0	3	3	0	3
	Hiến	0	2	2	0	2					
	Khánh	0	1	0	2	0					
	Tiến	0	3	3	0	3					
	Mean	**X_5 = 0.95**					**Y_5 = 1.80**				
6	Loan	1	1	0	0	0	1	1	0	2	0
	Tuyền	1	1	0	2	0					
	N.Trâm	1	1	0	2	0					
	Mean	**X_6 = 0.67**					**Y_6 = 0.80**				
7	Anh	0	0	0	0	0	0	1	2	0	2
	Sal	0	1	2	0	2					
	Đăng	0	1	2	0	2					
	Mean	**X_7 = 0.67**					**Y_7 = 1.00**				
8	Kỳ	0	2	0	0	2	0	2	2	0	2
	V.Liền	0	1	0	0	0					
	Tùng	0	1	0	0	0					
	Mean	**X_8 = 0.40**					**Y_8 = 1.20**				
9	M. Trí	0	1	1	0	1	0	1	1	0	1
	H. Em	0	1	1	0	1					
	Cảnh	1	1	1	0	1					
	Mean	**X_9 = 0.67**					**Y_9= 0.60**				
10	Hoàng	0	1	0	2	1	0	1	1	2	2
	Nhiệm	0	0	1	0	0					
	Trị	0	2	1	2	1					
	Mean	**Mean X_{10} = 0.73**					**Mean Y_{10} = 1.20**				

11	Khánh	0	2	1	1	1	1	2	0	0	2
	K.Trang	0	2	3	3	2					
	Tho	1	2	0	0	2					
	Mean	**X_{11} = 1.27**					**Y_{11} = 1.00**				
12	Anh	0	2	2	0	2	0	2	2	0	2
	Cần	0	2	2	0	2					
	Toàn	0	2	0	0	2					
	Mean	**X_{12} = 1.07**					**Y_{12} = 1.20**				
13	Ngân	0	1	0	0	0	0	1	0	0	1
	Đạt	1	0	0	0	1					
	Đ.Trang	0	1	0	1	0					
	Mean	**X_{13} = 0.33**					**Y_{13}= 0.40**				
14	An	0	2	2	0	1	0	2	2	0	0
	Phát	0	2	2	0	2					
	Thiệp	0	2	2	0	1					
	Mean	**X_{14} = 1.07**					**Y_{14} = 0.40**				
15	V.Trí	0	2	3	0	0	0	3	3	3	3
	Nhị	0	1	1	0	2					
	Qui	0	1	1	0	2					
	Mean	**X_{15} = 0.87**					**Y_{15} = 2.40**				
16	Hồng	0	2	2	0	0	0	1	0	0	0
	Hằng	0	2	2	0	0					
	Đươl	0	1	0	0	0					
	Mean	**X_{16} = 0.60**					**Y_{16} = 0.20**				
17	Lộc	0	2	0	1	0	0	1	0	2	0
	Tiên	0	1	0	2	0					
	Vinh	0	0	1	2	2					
	Mean	**X_{17} = 0.73**					**Y_{17} = 0.60**				

Table 5: Comparison the levels of using analogy according to the values of mean

Comparing X_i and Y_i	$X_i < Y_i$	$X_i > Y_i$
The number of groups	10	7

Comment: The results in Table 4 and Table 5 show that through working in group, the students increased the use of analogy in teaching contents in the topic "Coordinate method in space". However, the increase is not significant because Wilcoxon Signed-Rank Test gave the result (from http://www.socscistatistics.com/tests) as follows: the W-value is 53.5; the critical value of W for $N = 17$ at $p \leq 0.01$ is 23 (therefore, the result is *not* significant at $p \leq 0.01$). The result is not our expectation. In fact, we have hoped that through working in team, students would select the best strategy for teaching the above mathematics contents; analogy strategy is such a strategy.

Conclusion

Teaching mathematics with analogical reasoning is an effective learning - teaching tool as many international educators argued. The study showed that mathematics education students in Can Tho University did not prefer this strategy even though they have had opportunities to approach the strategy and especially for teaching topic "Coordinate method in space, the use of analogy with Coordinate method in plane analog can be considered as the best teaching strategy. This fact is a problem that university teachers need to reflect and to find out solutions.

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