Image Denoising Using Wavelet Thresholding Techniques

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***Abstract*— This research work proposes and explore different wavelets methods in digital image denoising. Using several wavelets threshold technique such as SUREShrink, VisuShrink, and BayesShrink in search for efficient image denoising method. In this paper, we extend the existing technique and provide a comprehensive evaluation of the proposed method. Wiener filtering technique is the proposed method which was compared and analysed, while the performance of all the techniques were compared to ascertain the most efficient method.**

***Keywords*—Wavelets, Image Denoising, Wiener Filter, Thresholding Techniques, Digital Image.**

I. INTRODUCTION

In many applications, image denoising is used to produce good estimates of the original image from noisy observations. The restored image should contain less noise than the observations while still keep sharp transitions. Wavelet transform, due to its excellent localization property, has rapidly become an indispensable signal and image processing tool for a variety of applications, including compression and denoising. Wavelet denoising attempts to remove the noise present in the signal while preserving the signal characteristics, regardless of its frequency content. It involves three steps: a linear forward wavelet transform, nonlinear thresholding step and a linear inverse wavelet transform. Wavelet thresholding proposed by Donoho is a signal estimation technique that exploits the capabilities of wavelet transform for signal denoising. It removes noise by killing coefficients that are insignificant relative to some threshold, and turns out to be simple and effective, depends heavily on the choice of a thresholding parameter and the choice of this threshold determines, to a great extent the efficacy of denoising. Researchers have developed various techniques for choosing denoising parameters and so far there is no unique or specific universal threshold determination technique. Denoising of natural images corrupted by noise using wavelet techniques is very effective because of its ability to capture the energy of a signal in few energy transform values.

II. THRESHOLDING

A. Introduction

The plot of wavelet coefficients in fig01 suggests that small coefficients are dominated by noise, while coefficients with a large absolute value carry more signal information than noise.

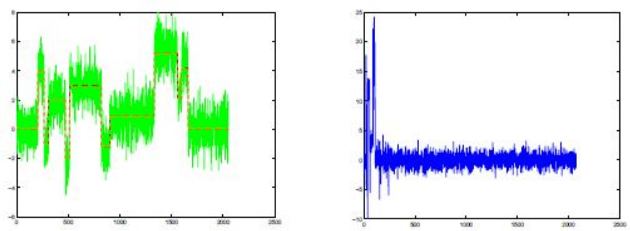


Fig01 A noisy signal in time domain and wavelet domain. Note the scarcity of coefficients.

B. Hard and Soft Thresholding

Hard and soft thresholding with threshold ¸ are defined as follows:

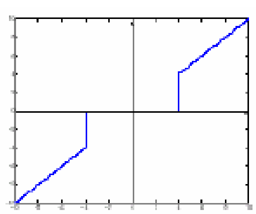
The hard thresholding operator is defined as:

D (U, λ) = U for all |U|> λ

= 0 otherwise

The soft thresholding operator on the other hand is defined as:

D (U, λ) = sgn(U)max(0, |U| - λ)



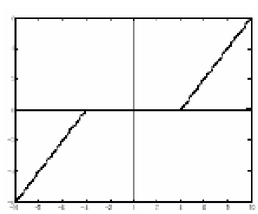


Fig02 Hard Thresholding Fig03 Soft Thresholding

C. Threshold Selection

Selection of thresholding is very important in denoising images. It is very sensitive because often the result of such output might be so close or nearly the same as that of the input with noise still present in the output signals.

The setup is as follows:

1. The original signals have length 2048.
2. We step through the thresholds from 0 to 5 with steps of 0.2 and at each step denoised the four noisy signals by both hard and soft thresholding with that threshold.
3. For each threshold, the MSE of the denoised signal is calculated.
4. Repeat the above steps for different orthogonal bases, namely, Haar, Daubechies.

Blocks

	Hard	Soft
Haar	1.2	1.6
Db2	1.2	1.6
Db4	1.2	1.6
Db8	1.2	1.8

Bumps

	Hard	Soft
Haar	1.2	1.6
Db2	1.4	1.6
Db4	1.4	1.6
Db8	1.4	1.8

Heavy Sine

	Hard	Soft
Haar	1.4	1.6
Db2	1.4	1.6
Db4	1.4	1.6
Db8	1.4	1.6

Doppler

	Hard	Soft
Haar	1.6	2.2
Db2	1.6	1.6
Db4	1.6	2.0
Db8	1.6	2.2

Fig03 Threshold with different denoising techniques in MSE

D. Comparison with Universal threshold

Numerous research has proven that universal thresholds give estimate for the soft threshold if the number of samples is larger since the threshold is optimal in the asymptotic sense.

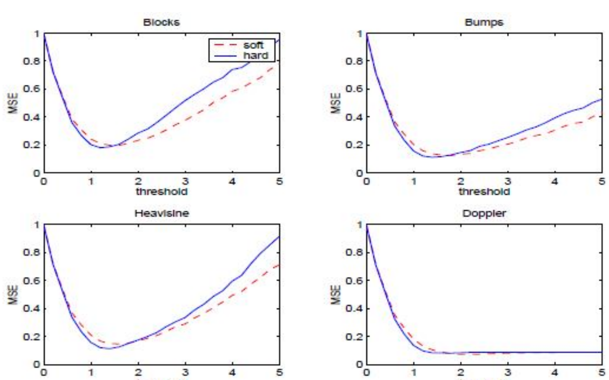


Fig04 Plot of MSE against Threshold values

III. WEINER FILTER

The Wiener filter purpose is to reduce the amount of noise present in a signal by comparison with an estimation of the desired noiseless signal. It is based on a statistical approach. Typical filters are designed for a desired frequency response. The Wiener filter approaches filtering from a different angle. One is assumed to have knowledge of the spectral properties of the original signal and the noise, and one seeks the LTI filter whose output would come as close to the original signal as possible. Wiener filters are characterized by the following:

1. Assumption: signal and (additive) noise are stationary linear stochastic processes with known spectral characteristics or known autocorrelation and cross correlation.
2. Requirement: the filter must be physically realizable, i.e. causal (this requirement can be dropped, resulting in a non-causal solution).
3. Performance criteria: minimum mean-square error.

IV. IMAGE DENOISING USING THRESHOLDING

An optimal threshold such that the mean squared error between the signal and its estimate is minimized. The wavelet decomposition of an image is done as follows: In the first level of decomposition, the image is split into 4 sub-bands, namely the HH, HL, LH and LL sub-bands. The HH sub-band gives the diagonal details of the image; the HL sub-band gives the horizontal features while the LH sub-band represents the vertical structures. The LL sub-band is the low resolution residual consisting of low frequency components and it is this sub-band which is further split at higher levels of decomposition. The different methods for denoising we investigate differ only in the selection of the threshold.

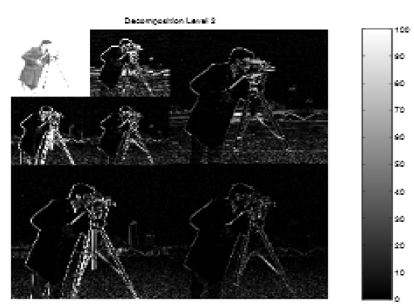


Fig05 2D – Cameraman Image

V. Image Denoising Using Wiener Filter

The Wiener filter in the wavelet domain removes the noise pretty well in the smooth regions but performs poorly along the edges. That is why it performs better on smooth images like Lena than on images with edges like the cameraman. For a noise variance of 400, the MSE was found to be 107.5 for the cameraman image and 80.5 for the Lena image.

Fig06 Denoised Image using the Wiener Filter in the Wavelet Domain. The noise variance was estimated using the MAD method.

Fig07 Denoised Image using the Wiener Filter in the Wavelet Domain with known noise variance

A. *Wiener filter in the wavelet domain vs Thresholding*

The Wiener Filtering in the wavelet domain was compared to the Thresholding methods and we saw that the Wiener filter outperforms both thresholding (see Figures 6 and 7) methods visually and in terms of MSE. More details were lost with the thresholding methods especially for the hard Threshold wherein the background was not well denoised. If the Wiener could be thought as another thresholding function, you get the intuition that it will perform better as its shape is smoother than the Hard and Soft thresholds.

Fig08 Denoised image obtained after thresholding the noisy wavelet coefficient with the Soft Threshold.

Fig09 Denoised image obtained after thresholding the noisy wavelet coefficient with the Hard Threshold.

B. *Wiener Filter in the wavelet domain vs Wiener Filter in the Fourier Domain*

It is clear that the method in the wavelet domain visually outperforms the global Wiener filter and Local Wiener Filter. However, the Wiener filter has a smaller MSE than the Wiener Filter in the wavelet domain and this is true with all values of noise variances we experimented with. This may be because the MSE is not always the best quantitative criterion.

VI. EXPERIMENTAL RESULT

The result of the experiment are tabled and graph below.

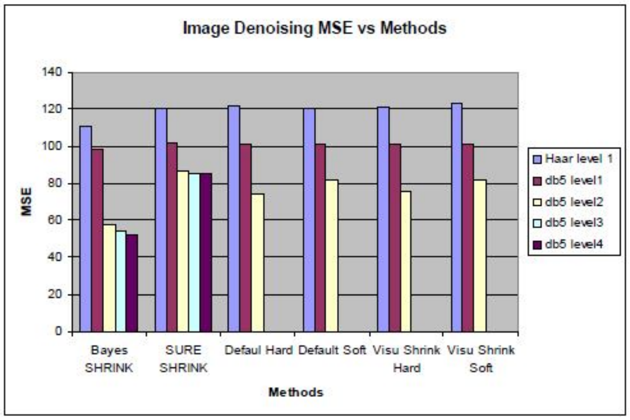


Fig10 Comparison of all methods based on MSE

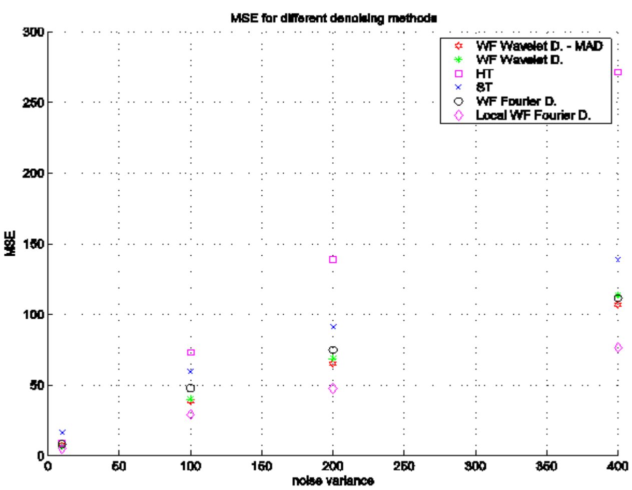


Fig11 Image Denoising in the Wavelet Domain using Wiener Filtering

More so, the Weiner's filter output result shown below:

Wavelet domain: WF vs Thresholding

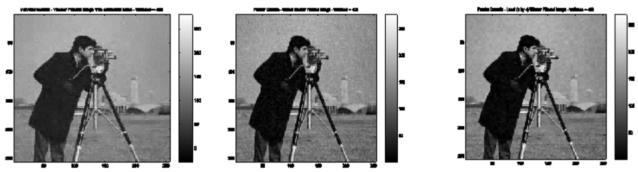
Weiner filter Soft thresholding Hard Thresholding

Fig12 Output of wiener and thresholding

TABLE I
MSE RESULT OF BETWEEN WIENER AND THRESHOLDING

Method	Wiener filter (Wavelet Domain)	Soft Thresholding	Hard Thresholding
MSE	110	140	175

WF: Wavelet Domain vs Fourier Domain



Weiner filter Global wiener Local wiener Fig13 Output of different wieners filter

TABLE 2
MSE RESULT OF DIFFERENT WIENERS

Method	Weiner filter (Wavelet Domain)	Global Wiener	Local Wiener
MSE	110	115	75

VII. CONCLUSION

Based on result of this research work, we concluded that Wiener filter in denoising images out-performed all the existing thresholding used in this paper work namely; SUREShrink, VisuShrink, and the BayesShrink. Wiener Filter in the Wavelet domain performs better than thresholding methods and Wiener Filter in the Fourier Domain. Mores so, Adaptive thresholding perform better than Universal thresholding. An important point to note is that although SUREShrink performed worse than BayesShrink, it adapts well to sharp discontinuities in the signal. Wiener needs some improve basically in some areas such as:

A. *Need for a better quantitative criteria*

B. *Improve denoising along the edges of the image*

VIII. ACKNOWLEDGEMENT

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