MATHEMATICAL MODELS OF THE RELATIONSHIP BETWEEN THE VOLUME AND SURFACE AREA OF AN EGG

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ABSTRACT

To establish the relationship between the surface area and volume of an egg has been a challenge to answer.

The volume and the surface area, of an egg are spherically related in the following equation:

$$\mathbf{A_s = kV^{2/3}} \quad \textbf{(1)}$$

where A_s, surface area; V, volume and k constant.

A more definite formula, ellipsoid, for the computation of the volume of the egg,

$$\mathbf{V = \pi LB^2/6(1+2/5c_2+1/5c_1^2+3/35c_2^2)} \quad \textbf{(2)}$$

where L, longitudinal length of the egg and B, breadth measured halfway between the poles of the egg.

Alternative formula for the volume:

$$\mathbf{V = \pi LB^2_{max}/6(1+2/5c_2 - 4/5c_1^2+3/35c_2^2)} \quad \textbf{(3)}$$

where B_{max} is the maximum breadth of the egg.

Experimental results using k=4.8359 in equation (1) resulted in a good estimate for the surface area. Further, a good estimate of the actual volume obtained in formulas (1) and (2) of the sample eggs.

The readers are encouraged to pursue similar studies using a variety of eggs from different breeds.

Keywords: Egg, Surface area, sphere, ellipsoid, volume

Section 1. Introduction

The methods for measuring egg size range from taking weights and linear measures to observing water displacement. One little-used approach is to estimate egg parameters from photographs. Estimating egg size from photographs was first pioneered by Paganelli et al. (1974), who traced the outlines of eggs using a planimeter to calculate volume and surface area. The technique described in this study is basically a modification of the above technique. While Paganelli et al used an automated computer analysis procedure to read data directly from digital photographs, eliminating the need to trace each egg, this study actually measured the egg by the length, breadth, maximum breadth, height.

Moreover, this present study aimed to illustrate the mathematical investigation of a real egg, using the basic principles of analytic geometry and calculus which are usually presented in classroom discussions.

It can be recalled that the surface area of an egg is occasionally desired or needed for computations of shell permeability or probable period of incubation. However, it is not easily measured directly, and cannot be computed from measurements of length and (Maximum) breadth without possible errors. An indirect method is to measure the volume of the egg, for instance, by total immersion in water or other liquid of known density, and hence to estimate the surface area, there is a relation between area and volume.

1.1 Statement of the Problem

This study aimed to discuss and find good estimations of surface area and volume of the egg on the basis of quantities that can be measured easily. Specifically, it aimed to:

1. carry out an experiment to determine the volume and surface area of a given hen's egg;
2. develop a mathematical model to compute the volume and surface area of a given egg; and
3. compare the experimental results with the results obtained via the mathematical model.

1.2 Significance of the Study

Mathematics and the real world are connected with each other in a rather direct way. This does not only increase the attractiveness of the work in the students' eyes, but it also brings them in touch with applications of mathematics on objects from daily life. Exploratory activities range from measurements, calculations, and figural analysis to functional analysis of models of physical objects.

The present study on the volume and surface area of an egg specifically looked into establishing a relationship between these two parameters. A sufficient knowledge about the volume and surface area allows one to determine some other specific characteristics of the egg like permeability and incubation period.

Results of this study could help facilitate research on real objects that are otherwise difficult to measure. For example, if students want to think of the shape of big objects like the main span of a suspension bridge and that of small objects like the shape of the plants cells plant, the size of the bacteria populations, and so on, they can use methods and techniques that are also applied by professionals in the field of space research, medicine, geography, and forensic.

1.3 Research Methodology

This study was carried out in analytic and experimental approaches. The analytic approach involves utilizing existing theories to establish the model to be used.

The experimental aspect of the study was used to generate data which served to verify the outputs of the model. Twenty-five randomly selected eggs bought from the local market were used to generate data on volume and surface area.

In finding the volume of the egg, the following was undertaken: the glass beaker was filled by with water until the water level indicated a volume of 200 ml with the egg below the waterline; the water was poured from the beaker into the calibrated measuring cylinder and the volume was noted; Calibrate the beaker by filling it without the egg until the water level of 200 ml was reached; the contents of the glass beaker were again poured into the calibrated measuring cylinder and the volume was noted. The difference of the measured volumes is the volume of the egg.

For the surface area, the following scheme was done: the egg was covered with pieces of kitchen foil such that few wrinkles as possible were formed and then the total area of the pieces of kitchen foil was measured.

1.4 Scope and Limitations

The subjects of the present study were limited to the commercial hen eggs available in the market. Sample eggs were randomly chosen and factors such as chicken breed, time of display and place of origin were not considered. The external volume was considered in effect the shell-part of the egg was included in the volume value. Surface area was determined by elementary measurement techniques. Some error in the actual measures might have occurred especially in the type of material used.

1.5 Definition of Terms

Breadth. It is the measure or dimension from side to side such as width.
Egg. It is the zygote, resulting from fertilization of the ovum for most birds and reptiles.
Ellipsoid. It is a type of quadric surface, a surface of an equation of second degree in three-dimensional Cartesian Coordinates that is a higher dimensional analogue of an ellipse.
Mathematical model. It is a representation of the essential aspects of an existing system which presents knowledge of that system in usable form.
Prolate spheroid. This refers to a spheroid that is *pointy* instead of *squashed*, i.e., one for which the polar radius is greater than the equatorial radius. A symmetrical egg (i.e., with the same shape at both ends) would approximate a prolate spheroid. A prolate spheroid is a surface of revolution obtained by rotating an ellipse about its major axis.
Space figure or three-dimensional figure. It is a figure that has depth in addition to width and height. Everyday objects such as a tennis ball, a box, a bicycle, and a redwood tree are all examples of space figures. Some common simple space figures include cubes, spheres, cylinders, prisms, cones, and pyramids. A space figure having all flat faces is called a polyhedron. A cube and a pyramid are both polyhedrons; a sphere, cylinder, and cone are not.
Sphere. It is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center. The distance from the center to the surface of the sphere is called its radius. Any cross-section of a sphere is a circle. If r is the radius of a sphere, the volume V of the sphere is given by the formula $V = 4/3 \times \pi \times r^3$. The surface area S of the sphere is given by the formula $S = 4 \times \pi \times r^2$.
Surface area. This refers to the total area of all the faces of the figure.

Volume. It is a measure of how much space a space figure takes up. It is used to measure a space figure just as area is used to measure a plane figure.

Section 2. RESULTS AND DISCUSSIONS

The surface area of an egg is occasionally desired or needed for computations of shell permeability or probable period of incubation. This chapter explores how the volume and the surface area of an egg can be related.

The determination of the volume of an egg can be made as easy as possible. One way is to measure the amount of water displaced when the egg is placed in a glass beaker.

In the absence of a calibrated measuring cylinder in which the egg fits, one can use a non-calibrated glass beaker and a calibrated measuring cylinder and employ the following procedure: filling the glass beaker with water until the water level indicates a volume of 200 ml with the egg below the waterline; pouring the water from the beaker into the calibrated measuring cylinder and noting down the volume; calibrating the beaker by filling it without the egg until the water level of 200 ml is reached; pouring again the contents from the glass beaker into the calibrated measuring cylinder and noting down the volume. The difference of the measured volumes is the volume of the egg.

An experimental determination of the surface area of the egg's surface without breaking the egg is not easy. One may think of the following method: covering the egg with pieces of kitchen foil such that few wrinkles as possible are formed and then measuring the total area of the pieces of kitchen foil. An alternative way could be: painting the egg and then rolling the egg on a piece of paper such that the area of the imprint corresponds with the surface area of the egg's surface. This method though remains tricky and this is the essence of the study comes from.

Experimental determination of quantities such as perimeter, surface area, and volume of an egg can be done, but in particular the determination of the surface area is difficult, error-prone and a real challenge. In the course of an investigation, students can consider the question whether it makes sense to repeat a measurement many times and compute the mean of the results. This is done in hopes of increasing the accuracy of results.

This study presents a more indirect determination of the surface area and the volume of an egg. Using an adequate mathematical description of the shape of the egg, the researcher establishes good estimations of surface area and volume of the egg on the basis of quantities that can be measured easily.

The descriptive quality of such mathematical model can be evaluated on the basis of the previously found experimental data.

2.1 Research Models

Since an egg is somewhat a sphere in form, the present study is based on the characteristics of a sphere, particularly its volume and surface area. One can, therefore, start with the formulas for finding the surface area and volume of the sphere.

Reviewing a classical result in solid geometry, known as the Cavalieri’s Theorem could be of help.

Theorem 1(CAVALIERI’S): If in two solids of equal altitude the sections made by planes parallel to and at the same distance from their respective bases are always equal, the volumes of the solids are equal.

Proof: Assuming a plane surface of area $\boldsymbol{B}$ is moved to a distance h in a direction perpendicular to itself, one can generate a solid of volume Bh; this solid can be called an *elemental solid.*

Let the two solids P and Q rest on the same horizontal planes, and common altitude H be divided into **n** equal parts each equal to **h**. Through the points of division of the altitude pass planes parallel to the horizontal plane of the basses. These planes will intersect the two solids in sections $S_1, S_2, \ldots S_{n-1}$ parallel to the basses and distant h apart.

On the basses S_0 of P and Q, and on each of the parallel sections $S_1, S_2, \ldots, S_{n-1}$ as basses, elemental solids of height h are constructed. Then the volume of each elemental solid is h times the area of its base S_i.

Denoting P' the volume of the elemental solids belonging to solid P, and Q' the volume of the elemental solids belonging to Q, and since the bases of corresponding elemental solids in P and Q are equal by hypothesis, and they have the same altitude h, one finds that their volumes are equal. Therefore, since the number of elemental solids is the same in P and Q, one can have **P' = Q'**. Suppose the number of subdivision n is increased indefinitely, thereby increasing the number of elemental solids indefinitely and decreasing their altitudes h in a corresponding manner, one can note that P' will approach P as a limit, and Q' will approach Q as a limit. But P' and Q' are two variables which are always equal. Hence by a theorem on limits which states that *If two variables are always equal and each approaches a limit, the limits are equal, one can come up with* **P = Q**. $\blacksquare$

Using Cavalieri's Theorem, one can derive the formula for the surface area and volume of the sphere.

Theorem 2. The surface area A_s of a sphere is given by $\mathbf{A_s = 4\pi r^2}$.

Proof: Considering the hemisphere cut from the sphere of center O and radius r, one can pass two planes distant y apart and parallel to the base of the hemisphere, cutting the hemisphere in two circles of radii r_1 and r_2. If one assumes ***arc DF*** = ***chord DF*** = **l** (the error introduced by taking ***arc DF = chord DF*** may be made as small as one pleases by taking y sufficiently small, the surface of the hemisphere included between these planes is equal to the lateral surface of the inscribed frustum of a right circular cone. This frustum has a slant height l, and an altitude y, and base radii r_1 and r_2. Its lateral surface is, by **(1)** $\mathbf{A_s = (2\pi r_1 + 2\pi r_2)l/2}$ or $\mathbf{A_s = \pi(r_1+r_2)l}$.

Let B be the midpoint of ***chord DF***. Then OB is perpendicular to ***chord DF*** and within the limits of the approximation, it is equal to the radius r of the sphere. One can therefore, denote the radius AB of the mid-section of the frustum by r_t since r_t is the mid-section of a trapezoid, **(2)** $\mathbf{r_t = (r_1+r_2)/2}$ Since angles AOB and FDE have their sides respectively perpendicular, one can observe that they are equal and right triangles AOB and FDE are similar. Therefore, **(3)** $\mathbf{r_t/r = y/l}$ or $\mathbf{r_t = ry/l}$.

Substituting in equation (3) the value of r_t from equation (2), one gets $\mathbf{(r_1 + r_2) / 2 = ry/l}$. Substituting this value of $(r_1 + r) / 2$ in formula (1), one obtains $\mathbf{A_s = 2\pi ry}$. By thinking of a sphere being formed by an indefinitely large number of these frustum, the sum of this altitudes is $2r$, it is evident that the formula for the surface of a sphere of radius r is $\mathbf{A_s = 2\pi r(2r)}$ **or** $\mathbf{A_s = 4\pi r^2}$. $\blacksquare$

Theorem 3. The volume V of a sphere is given by $\mathbf{V = 4/3\pi r^3}$.

Proof: Considering the hemisphere cut from the sphere of center O and radius r, one can compare this hemisphere with the solid which results from removing a right circular cone of base radius r and altitude r from a right circular cylinder of the same base and altitude.

Two solids are placed so that their basses lie on the same plane. A plane is passed parallel to and distant y from the bases, cutting the hemisphere in a small circle A and the other solid in a section A' (area bounded in a two concentric circle). The radius of the circle A is denoted by r_a, the inner radius of section A' by x (the outer radius of section A' is obviously r), and (1) $\mathbf{A = \pi r_a^2}$ and **(2)** $\mathbf{A' = \pi(r^2 - x^2)}$ are given.

Since the legs of right triangle ***CDE*** are each r, $\boldsymbol{\theta = 45^\circ}$. Whence $\boldsymbol{x = y}$. Applying the Pythagorean theorem to right triangle ***OO'B***, one has $\mathbf{r_a^2 = r^2 - y^2}$. Substituting this value of r_a^2 in (1), and putting x = y in (2), one obtains

$$\mathbf{A = \pi(r^2 - y^2)} \text{ and } \mathbf{A' = \pi(r^2 - y^2)}$$

Hence, **A = A'.**

Since the altitude of each solid is equal to ***r*** and since A = A', it follows from

Cavalieri's theorem that the volumes of the two solids are equal. But,

denoting the volume of the constructed solid by V1, one has

$\mathbf{V_1}$ **= volume of cylinder–volume of cone** or $\mathbf{V_1 = (\pi r^2)r - 1/3(\pi r^2)r = 2/3\pi r^3}$**.**

Therefore, the volume of the hemisphere is $\mathbf{V_1 = 2/3\pi r^3}$.

Hence, the volume of a sphere of radius r is $\mathbf{V = 4/3\pi r^3}$**.** $\blacksquare$

Consider a solid with a surface area $\mathbf{A_s}$ and volume **V**. The following theorem establishes a relation between $\mathbf{A_S}$ and **V**.

Theorem 4. Let $\mathbf{A_s}$ and **V** denote the surface area and volume of a sphere, respectively, then $\mathbf{A_s = kV^{2/3}}$ where **k** is a "dimensionless" constant.

Proof*:* Using Theorems 2 and 3, $\mathbf{V = 4/3\pi r^3}$ and $\mathbf{A_s = 4\pi r^2}$

One can try to express A_s in terms of V.

$$\mathbf{A_s = 4\pi r^2 = 4/3\pi r^3\ (3/r) = V\ (3/r)}$$

Then, $\mathbf{(4\pi r^2 = V\ (3/r))^{2/3}}$

$$\mathbf{(4\pi r^2)^{\wedge 2/3} = V^{2/3}(3/r)^{2/3}}$$

$$\mathbf{\{(4\pi r^2)/\ (3/r)\}^{2/3} = V^{2/3}}$$

$$\mathbf{(4/3\pi r^3)^{2/3} = V^{2/3}}$$

$$\mathbf{(4/3\pi)^{2/3} r^2 = V^{2/3}}$$

then multiply the equation by $\underline{\mathbf{4\pi}}$ / $\mathbf{(4/3\pi)^{2/3}}$. One obtains $\mathbf{4\pi r^2 = 4\pi\ /\ (4/3\pi)^{2/3}\ V^{2/3}}$**.**
But one knows that $\mathbf{4\pi r^2 = A_s}$**.** So, $\mathbf{A_s = 4\pi\ /\ (4/3\pi)^{2/3}\ V^{2/3}}$ **or** $\mathbf{A_s = kV^{2/3}}$ where the value of **k=4.8359**. This is true for a sphere. $\blacksquare$

Since an egg is not perfectly spherical in form, the value for k varies according to the shape of the egg being considered. One can now turn to approximate the volume of an egg.

Model 1. Equatorial Length Approach

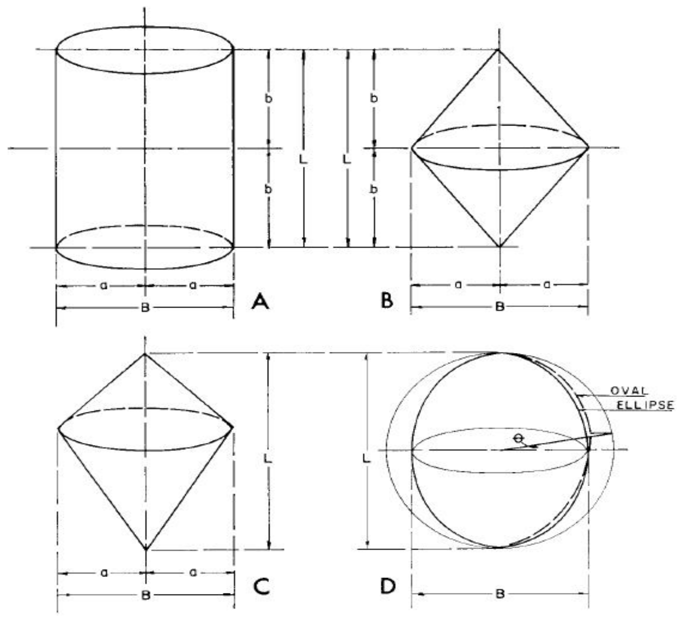

Fig.1. A. Cylinder. B. Bicone C. Two Cones D. A Circle Circumscribes an Ellipse

The parametric equation of the longitudinal section of an egg may be taken (Fig.1D) as (1) $\mathbf{y=b \sin\theta}$ and (2) $\mathbf{x = a \cos\theta(1 + c_1 \sin\theta + c_2 \sin^2\theta + etc)}$,
where **θ** is the "eccentric angle," ***a*** is the semi diameter at the true equator (i.e. halfway between the two ends of the egg), ***b*** is the half-length of the egg, c_1 and c_2 are coefficients that vary from egg to egg and have to be found experimentally, and the terms labeled "+ etc." can be usually neglected c_1 and c_2 are usually quite small, so that c_1^2 and c_2^2 and $c_1 c_2$ can be neglected.

Slicing the egg parallel to the equator (perpendicular to the long axis of the egg) into small thicknesses dy, gives one various elements of volume $\mathbf{dV = \pi x^2 dy}$ and total volume of the egg is (3) $\mathbf{V=\int \pi x^2 dy}$ which is evaluated from $-\pi/2$ to $\pi/2$. Ignoring terms that include negligible coefficients, one has
$\mathbf{x^2 = a^2\cos^2\theta(1+2c_1\sin\theta+2\ c_2\sin^2\theta)}$ and $\mathbf{dy = b \cos \theta\ d\theta}$.

So, (4) $\mathbf{V = \pi a^2 b \int \cos^3\theta(1 + 2c_1\sin\theta + 2\ c_2\sin^2\theta)\ d\theta}$ is evaluated from $-\pi/2$ to $\pi/2$. The complete integral from $-\pi/2$ to $\pi/2$ of the middle term, therefore vanishes (being $-\cos^4\theta$) and the integral reduces to (4a) $\mathbf{V = \pi a^2 b \int (\cos^3\theta + 2c_2\sin 3\theta\sin^2\theta)\ d\theta}$ and by writing $\mathbf{\cos^3\theta=\cos\theta(1-\sin^2\theta)}$, this integrates to
(5) $\mathbf{V=4\pi/3[(a^2b)\ (1+2/5c_2)]}$

If the length of the egg is $\mathbf{L = 2b}$ and its equatorial (not necessarily maximum) breadth is $\mathbf{B = 2a}$ this equation takes form (5a)$\mathbf{V = \pi/6[(LB^2)\ (1 + 2/5c_2)]}$.

If c_2 is zero this reduces to $\mathbf{V = \pi/6[(LB^2)]}$, the volume of an ellipsoid of revolution, and it does not depend on c_1 at all, provided we were justified in assuming that c_1 is comparatively small and c_2 is negligible. c_2 can be either positive or negative. With most species and individual parents, c_2 is negative, so the volume of the egg is less than the volume of the circumscribing ellipsoid.

The effect of using $\mathbf{B_{max}}$ instead of $\mathbf{B_{equatorial}}$ in the parametric equation $\mathbf{y = b \sin\theta}$ and $\mathbf{x = a \cos\theta(1 + c_1 \sin\theta + c_2 \sin^2\theta)}$, is the maximum value of x that is obtained when dx/dy is zero or when $dx/d\theta = 0$.

Let $\mathbf{\theta_m}$ be the value of $\mathbf{\theta}$ that makes $\boldsymbol{x}$ a maximum. If $\boldsymbol{c_1 = c_2} = 0$, the equation of an ellipse, one gets (6) $\mathbf{dx/d\theta = -a \sin \theta}$, **and this is zero when** $\mathbf{\theta = 0}$**.** This is a correct solution.

Now let $\boldsymbol{c_2} = \mathbf{0}$ but let $\boldsymbol{c_1}$ be non-zero. Then, recalling that $\mathbf{\sin \theta_m}$ is assumed small and therefore that $\mathbf{\cos \theta_m}$ is very near unity, one gets

(7) $\mathbf{\sin \theta_m = [- 1 + sq(1 + 4c_1^2)]/2c_1}$

Recalling that $\boldsymbol{c_1}$ is much less than unity, the square root term is very nearly $\mathbf{(1 + 2c_1^2)}$, so that (7a) $\mathbf{\sin \theta_m = c_1}$ is very nearly.

If $\boldsymbol{c_1}$ and $\boldsymbol{c_2}$ are both non-zero, but $\cos \theta_m$ is very near unity ($\sin \theta_m$ being small), one gets a cubic equation for $\sin \theta$ as follows:

(8) $\mathbf{c_2\sin^3\theta_m + c_1\sin^2\theta_m + (1 - 2c_2)\sin\theta_m - c_1 = 0}$

For small values of $\sin \theta_m$, c_1, and c_2 then from (7a), then the value of the diameter is (9) $\mathbf{B_{max} /B = (1 - c_1^2/2)(1 + c_1^2)}$, then (10) $\mathbf{B_{max}/B = 1 + ½ c_1^2}$.

so one has derived the formula for volume:

(11) $\mathbf{V = \pi LB^2/6 (1 + 2/5 c_2 + 1/5 c_1^2 + 3/35 c_2^2)}$,

<u>Model 2. Ellipsoid Approach</u>

There is an evident shortfall of the first model. In most egg samples, the widest breadth is not halfway between the poles. This is improved by the model proposed by Tatum.

Model 1 expresses the volume, **V**, in terms of the length, **L**, and the breadth, **B**, measured half way between the poles of the egg. It generally relied on the assumption that the shape of an egg can be described by the revolution about its long axis of an oval figure whose parametric equations are (1) $\mathbf{y = b \sin\theta}$ and

(2) $\mathbf{x = a \cos\theta(1 + c_1 \sin\theta + c_2 \sin^2\theta)}$, where $\mathbf{c_1}$ and $\mathbf{c_2}$ are coefficients representing the departure of the oval from an ellipse. In particular, c_1 represents a departure from symmetry, being zero for a symmetric egg.

The said formula for **V** was developed to first order in $\boldsymbol{c_1}$ and $\boldsymbol{c_2}$(In this order it is independent of $\boldsymbol{c_1}$). The formula for $\mathbf{B_{max} /B}$ was developed to second order in $\boldsymbol{c_1}$ and $\boldsymbol{c_2}$(In this order it is independent to $\boldsymbol{c_2}$). So in order to express **V** directly in terms of **L** and $\boldsymbol{B_{max}}$ both parts must be carried in the same order where c_1 and c_2 are carried to second order.

For the volume in terms of the length and equatorial breadth in the second order in $\boldsymbol{c_1}$ and $\boldsymbol{c_2}$, one has: (3) $\mathbf{V = \pi LB^2/6 (1 + 2/5 c_2 + 1/5 c_1^2 + 3/35 c_2^2)}$,

In retracing Model 1 $\mathbf{B_{max}/B}$, we can expect some small errors and Model 1 equation (8) should read (4) $\mathbf{3c_2 \sin^3\theta_m + 2c_1\sin^2\theta_m + (1\text{-}2c_2)\sin \theta_m - c_1 = 0.}$

Enforcing $\mathbf{B_{max}/B}$ correct to second order such that $\mathbf{\sin \theta_m}$ must also be taken to second order, one can come up with the following second order expression:

(5) $\mathbf{\sin \theta_m = c_1 + 2c_1c_2}$ the resulting expression for $\mathbf{B_{max}}/\boldsymbol{B}$ to second order is

(6) $\mathbf{B_{max}/B = 1 + ½ c_1^2}$.

One can express **V** directly in terms of **L** and $\mathbf{B_{max}}$ from equations (3) and (6) to second order in $\boldsymbol{c_1}$ and $\boldsymbol{c_2}$ which one can call as Model 2:

(7) $\mathbf{V = \pi LB^2_{max}/6 \ (1 + 2/5 c_2 - 4\backslash 5 c_1^2 + 3\backslash 35 c_2^2)}$,

In Model 2, volume is determined by taking four measurements: length ***L*,** breadth **B,** maximum breadth $\mathbf{B_{max}}$, and height **H** as indicated by Fig.2.

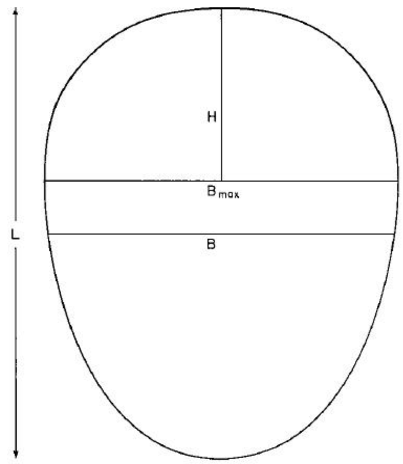

Figure 2. The Egg.

The coefficient c_1 is determined from the equation (6). The value of **$\sin\theta_m$** is given by (8) **$\sin\theta_m = 1 - 2H/L$** so that c_2 can be determined from the equation (5). The volume can now be calculated from equation (7).

2.2 Experimental Simulations

The accuracy of the two models using the 25 egg samples is now determined. Table 1 shows how the actual volume of the egg samples differed from the computed volumes using the two models.

As seen from the table below, the actual volume did not significantly differ from the projected volume using either of the models. The computed F-value of 2.834 gave a p-value of 0.065 which is above the alpha level of 0.05.

Thus, it can be inferred that either models give a close estimate of the volume of the eggs.

Table 1. Analysis of Variance between the actual volume and the model outputs.

Source of Variation	Sum of Squares	df	Mean Squares	F
Model	1001	2	500.4	2.834ns
Error	12717	72	176.6	
Total	13717	74		

Table 2. Correlations of actual volume with the model outputs.

	r - value
Model 1	0.964**
Model 2	0.966**

** - significant at alpha = .01

This is further affirmed by the very significant correlation among the values in the three measurements as indicated in Table 2. As seen in Table 2, the correlation between the actual volume and the expected volume based from Model 2 is 0.964 which is highly significant. This indicates a very high accuracy of the model in approximating the volume of an egg. Similarly, one finds the same observations with Model 1.

Table 3. Analysis of Variance between the actual surface area and the model outputs

Source of Variation	Sum of Squares	df	Mean Squares	F
Model	622	2	311.0	2.708ns
Error	8271	72	114.9	
Total	8893	74		

Table 3 shows how the actual surface area of the egg samples differed from the computed surface area using the two models. As shown in the table, the actual surface area did not significantly differ from the projected surface area using either of the models. The computed F-value of 2.708 gave a p-value of 0.073 which is above the alpha level of 0.05. Thus, it can be inferred that either models gives a close estimate of the surface area of the eggs.

Table 4. Correlations of actual surface area with the model outputs.

	r - value
Model 1	0.963**
Model 2	0.965**

** - significant at alpha = .01

This is further affirmed by the very significant correlation among the values in the three measurements as indicated in Table 4. Moreover, according to Table 4, the correlation between the actual surface area and the expected surface area based from Model 2 is 0.963 which is highly significant. This indicates a very high accuracy of the model in approximating the surface area of an egg. Similar observations were seen with Model 1.

Section 3. SUMMARY AND RESEARCH DIRECTION

Using the sphere as a basis, the volume and the surface area of an egg are related in the following equation

$$\mathbf{A_s = kV^{2/3}} \qquad \mathbf{(1)}$$

where A_s and V denote the surface area and volume, respectively, while k is a constant. The value of k varies depending on how the egg deviates from or resembles the shape of the sphere.

Treating the egg as an ellipsoid, a more definite formula for the computation of the volume of the egg is

$$\mathbf{V = \pi\ LB^2/6\ (1 + 2/5\ c_2 + 1/5\ c_1^2 + 3/35\ c_2^2)} \qquad \mathbf{(2)}$$

where L is the longitudinal length of the egg and B is breadth (measured halfway between the poles of the egg).

Observing that the maximum breadth is not necessarily halfway between the poles, one can have the following alternative formula for the volume:

$$\mathbf{V = \pi\ LB^2_{max}/6\ \ (1 + 2/5\ c_2 - 4/5\ c_1^2 + 3/35\ c_2^2)} \qquad \mathbf{(3)}$$

where B_{max} is the maximum breadth of the egg.

Experimental results indicate a good estimate of the actual volume of the sample eggs considered in the study. Using analysis of variance, it was revealed that the actual volume did not significantly differ from the volumes obtained in formulas (1) and (2).

A consequent approximation of the surface area using k=4.8359 in equation (1) resulted in a good estimate for the surface area of the sample eggs.

With these results, the researcher encourages the readers to pursue similar studies directed to further tighten the relationship between the volume and surface area of an egg. The author suggests the use of a variety of eggs from different breeds.

Likewise, modeling problems on natural occurrences should be conducted to make mathematics more appealing and practical especially to non-mathematics practitioners.

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APPENDIX A. MEASUREMENT DATA OF THE 25 SAMPLE EGGS

Egg No.	L	B	Bmax	H	C_1	Sinθm	C_2
1	5.6	3.8	3.9	1.6	0.229416	0.428571	0.022845
2	5.7	4	4.1	1.7	0.223607	0.403509	0.020114
3	5.6	3.8	4	1.4	0.324443	0.5	0.028479
4	5.7	4	4.1	1.6	0.223607	0.438596	0.024037
5	5.6	4	4.3	1.5	0.387298	0.464286	0.014909
6	5.7	4.1	4.4	1.5	0.382546	0.473684	0.017432
7	5.7	4.3	4.4	1.7	0.215666	0.403509	0.020256
8	5.7	4.4	4.5	1.5	0.213201	0.473684	0.027768
9	5.9	4.4	4.6	1.6	0.301511	0.457627	0.023535
10	5.8	4.5	4.7	1.7	0.298142	0.413793	0.01724
11	6	4.3	4.7	1.8	0.431331	0.4	-0.00676
12	5.8	4.6	4.9	1.7	0.361158	0.413793	0.009505
13	6	4.3	4.5	1.9	0.304997	0.366667	0.009405
14	5.8	4.7	4.9	1.8	0.29173	0.37931	0.012775
15	5.9	4.8	5	1.7	0.288675	0.423729	0.019493
16	6	4.8	4.9	1.9	0.204124	0.366667	0.016589
17	6.1	4.8	5	2	0.288675	0.344262	0.008023
18	6.1	4.7	5	1.9	0.357295	0.377049	0.003529
19	6.2	4.8	5	1.9	0.288675	0.387097	0.014206
20	6.4	4.8	5	1.9	0.288675	0.40625	0.01697
21	6.4	5	5.2	2.3	0.282843	0.28125	-0.00023
22	6.6	4.8	5	2.4	0.288675	0.272727	-0.0023
23	6.6	4.9	5.1	2.3	0.285714	0.30303	0.002474
24	6.5	5.2	5.3	2.1	0.196116	0.353846	0.015467
25	6.8	5	5.2	2.3	0.282843	0.323529	0.005754

APPENDIX B. VOLUME AND SURFACE AREA MEASUREMENT DATA OF THE 25 SAMPLE EGGS

Egg No.	V(Model 1)	Vwater	V(Model 2)	A(model 1)	Awater	A(Model 2)
1	44.6	45	42.3	59.47	61.18	59.52
2	50.2	46	47.8	64.37	62.08	64.42
3	46.9	45	42.3	59.82	61.18	60.01
4	50.2	46	47.7	64.44	62.08	64.48
5	54.2	46	46.9	63.9	62.08	64.4
6	57.8	47	50.2	66.87	62.98	67.36
7	57.8	48	55.2	70.87	63.87	70.91
8	60.4	49	57.8	73.22	64.76	73.25
9	65.4	50	59.8	75.13	65.63	75.31
10	67.1	59	61.5	76.39	73.29	76.58
11	69.4	59	58.1	73.19	73.29	74.19
12	72.9	61	64.3	78.65	74.94	79.12
13	63.6	59	58.1	73.39	73.29	73.61
14	72.9	60	67.1	80.82	74.12	81.02
15	77.2	62	71.2	84.23	75.75	84.41
16	75.4	63	72.4	84.79	76.57	84.83
17	79.8	63.5	73.6	85.84	76.97	86.03
18	79.8	62.5	70.6	83.55	76.16	84.05
19	81.2	66.5	74.8	86.93	79.38	87.12
20	83.8	66	77.2	88.86	78.98	89.05
21	90.6	71	83.8	93.34	92.92	93.57
22	86.4	71.5	79.6	90.2	83.31	90.44
23	89.9	72	83	92.83	83.69	93.05
24	95.6	74	92	99.45	85.24	99.48
25	96.3	73	89	97.36	84.47	97.57

APPENDIX C. THE 25 SAMPLE EGGS AND RESEARCHER'S WORKING TABLE