Different Theoretical Perspectives on Specific Learning Difficulties in Mathematics. Implications for Special Educational Intervention and for Everyday School Practice": An overview study

Dr. Eleni Bonti, Afroditi Kamari, Dr Georgios Kougioumtzis,
Dr. Antonis Theofilidis & Dr. Maria Sofologi

ABSTRACT

As our society becomes more and more dependent on high levels of computer-based technology, it becomes increasingly important that children should grow up with basic competence and familiarity with numbers. Yet, a significant amount of evidence suggests that many children encounter major problems in learning mathematics. But why do children find arithmetic so difficult? In the present study an attempt is being made to explore the ways in which different psychological perspectives have been used to explain the nature of children' s in mathematics and the possible reasons that might account for children' s difficulties in this area. More specifically, the study initially focuses on the distinction often made between those who favor the mechanistic-procedural skills involved in acquiring numeracy, versus those who place the emphasis upon the conceptual understanding of mathematics. The three psychological perspectives which dominate in the relevant literature, that is, the 'constructivist', the 'social constructivist' and the 'information-processing' theories are also considered with respect to their implications for understanding children' s mathematical development and difficulties with number. Finally, the several ways in which the above theories might be interpreted in order to assist educators in meeting their pupils' learning difficulties in the area of mathematics, are also discussed.

Keywords: Specific learning difficulties, Mathematics, number, psychological perspectives, intervention issues, mechanistic versus conceptual understanding of math

Introduction

'Mechanistic' versus 'Conceptual' understanding of Mathematics

It is very often the case that children's difficulties with numeracy are a result of their negative emotional reactions towards the particular task, which is viewed as a set of meaningless symbols, rules and formulas, that "you need to learn" in order to be able to carry out lists of fractions and to solve mathematical problems, which have nothing to do with everyday life situations (Fisher, 1990). Evidence suggests that these negative feelings are often created by the actual ways in which children are being taught mathematics (Quine, 2009).

Research into the psychology of learning mathematics has made a distinction between two different manners by which mathematics can be learned. These are: The 'instrumental-mechanical' and the 'relational-conceptual' types of knowledge. The first can be gained by learning rules (algorithms) and being able to apply them on across different practical circumstances. As Fischer (1990: 208) argues, "the trouble with rules is that they are easily forgotten". By contrast, the second kind of knowledge implies that the learner has understood the reasoning behind the rules and this understanding can be gained if the child has thought through and has flexibly applied those rules in different contexts (Gomez, Piazza, Jobert, Dehaene-Lambertz, Dehaene, & Huron, 2015).

As Daniels and Anghilieri (1995) state, for many generations, school mathematics was taught as a mental discipline, involving indisputable facts and standard procedures, thus being in line with the 'mechanistic' approach. Moreover, at that time, mathematics was only a discipline for the privileged few and was being learned for its use in a non-technological society, or simply for its aesthetic value and beauty. However, it seems that the current educational system has recognized the changing needs of society and has appreciated the need for pupils to have a positive attitude towards the subject, and to have confidence in using mathematical skills in everyday problem-solving (Kakia & Kougioumtzis, 2016). This alternative way of viewing mathematics has been highlighted in the Cockroft Report (1982), which, according to the above researchers, “has been a cornerstone for establishing the principles for teaching mathematics today” and is even more explicit within the National Curriculum Draft Proposals (Dearing, 1994), which focus on pupils' ability to use and apply mathematics on practical tasks and in real-life problems and which propose entitlement to a broad and balanced curriculum and appropriate programs of study, matched to the needs of each individual (Bartelet, Ansari, Vaessen & Blomert, 2014).

On the other hand, other writers argue that there is considerable evidence suggesting that the school mathematics curriculum, worldwide, is still essentially technique-oriented and based on procedures, methods, skills, rules and algorithms (Willis, 1990). It seems that the reason for this contradiction lies in the various ways in which educationalists and practitioners over the years, have interpreted and utilized the different psychological perspectives and conceptualizations concerning the development, learning and teaching of mathematics (Bonti, 2013).

An overview of those psychological perspectives follows, with respect to the views of the 'key-people', who have contributed to their development. The ways in which those theories have influenced the teaching and learning practice, is also considered (Dowker, 2017).

The 'Behaviorist' Perspective

The basic theory, which has accounted, until recently, for the majority of mathematics teaching is the 'mechanistic' or 'procedural' approach (Daniels & Anghilieri, 1995), which has its origins in the way behaviorists, such as Thorndike (1913) analyzed human behavior in terms of stimulus and response. Applying this theory to arithmetic learning, Thorndike advocated establishing and strengthening associations through 'drill and practice'. Thus, according to Thorndike (1922), learning arithmetic is essentially a process of forming bonds between a stimulus (eg. 'two and two') and a response ('four'). As with other aspects of behaviorist theory, practice followed by reward is considered the most likely mechanism by which the bonding would take place (in: Huges, 1986). In other words, the behaviorist perspective takes little account of the child' s level of conceptual development and, if translated into practice, the teachers' task is simply to provide the child with positive reinforcement and sufficient practice for the correct bonds to be learned.

The “Constructivist” Perspective

Given the educators dissatisfaction with traditional methods, such as the 'transmission' model essentially advocated by Thorndike, it is not difficult to see how appealing Piaget's theory about how children learn mathematics, might have been. Piaget was the most fundamental contributor to the theory of 'constructivism', whose conceptualizations about children' s development, provided the theoretical framework that has influenced mathematics teaching and curriculum from the 1960' s.
According to Piaget (1952), children construct their own knowledge through interactions with the environment, developing a cognitive structure by way of organizing experiences (in: Daniels & Anhilieri, 1995). Of central theoretical importance to Piaget, is that cognitive development is a coherent process of successive qualitative changes of cognitive structures ('schemata') from birth to adolescence, which take place in a definite, inevitable sequence of maturational steps or stages.

Transition from stage to stage occurs as a result of 'assimilation', whereby children incorporate new experiences into existing structures, by acting on their environment, and 'accommodation', through which, these experiences are modified to incorporate new experiences (Piaget & Inhelder, 1966). In addition to his stage theory of development, Piaget had distinctive views on the way in which mathematics is learned and, therefore, on how it should be taught (Alabdulaziz & Higgins, 2017).
For Piaget, number is a 'mental structure' that each child constructs out of natural ability to think, rather than learns from the environment. 'Physical' knowledge, which is derived directly from actions on objects and 'logico-mathematical' knowledge, in which the objects serve as a medium for permitting the construction to occur, are the two major types, or poles, of knowledge distinguished by Piaget (Gallagher & Reid, 1981, in: Wadsworth, 1989). Moreover, according to Piaget, number is a synthesis of two kinds of relationships the child creates among objects. One is 'order' and the other is 'hierarchical inclusion'. Thus, when a child is presented with, for example, eight objects, s/he can quantify the collection numerically only if s/he can put all the objects into a single relationship, thus synthesizing order and hierarchical inclusion. His basic argument, however, was that the 'pre-operational' child (i.e. the child being in the second stage of development, which covers the period from roughly 18 months to 7 years), lacks the kind of 'mobility' of thought that would allow him or her to put all kinds of contents (objects, events and actions) into all kinds of relationships.

Another characteristic most children below 7-8 years of age, lack, according to Piaget, is 'reversibility'. Reversibility refers to the ability to mentally carry out opposite actions simultaneously. Thus, lack of mobility in children' s thought, accounts for their inability to perform numerical tasks (Kamii & De Clark, 1985).

In order to further support these arguments, Piaget carried out a number of experiments, involving 'class-inclusion' and 'conservation' problems. The class-inclusion problems were intended to be a test of the child' s ability to compare a set with a subset of itself or a whole with one of its parts. His argument, in brief, was that the pre-operational child is simply incapable of comparing a set with one of its subsets. According to Piaget, the child can only attend either to the set or to the subset, but can never take account of both, at the same time. Moreover, Piaget claimed that understanding class-inclusion is an essential prerequisite for understanding addition and subtraction. The number conservation task was again used to show that children below the age of 7-8 years are not able to conserve number. That is, they respond as if they believe that changing the length of a row containing the same number of beads with a second row, also changes its numerosity. Piaget, again, maintains that if children cannot conserve number, then they are not yet 'ready' to start on school arithmetic (Huges, 1986).

The main implication of the above conceptualizations, regarding children' s development of number, for teaching, is that arithmetic is something children reinvent and not something that has to be transmitted to them. Moreover, Piaget' s concept of 'readiness' could be used to explain why some children experience difficulties with mathematics. In other words, as Kamii & DeClark (1985: 35) state, “if math is so difficult for so many children, it is generally because it is imposed too early, and without adequate awareness of how children think and learn”. If we accept this view, then the teacher' s role should be regarded as being intellectually non-interventionist and should only be restricted in providing a rich and stimulating environment, which can be actively used by the children for constructing their knowledge (Bonti, 2013).

However, even though Piaget' s ideas have influenced early childhood education to a great extent, they have also attracted increasing amounts of criticism within the area of developmental psychology (Donaldson, 1978. Gelman & Gallistel, 1978). In particular, there is now considerable evidence that children starting school are by no means, as limited in their number concepts, as Piagetian theory maintains (Huges, 1991, in: Light et al., 1991). All the above psychologists share the belief that children' s failure on various mathematical tasks, might be due to factors other than the lack of ability or readiness.

A number of studies carried out by McGarrigle and Donaldson (1974), provide several interesting findings in sharp contrast to those generated by Piaget, concerning both his class-inclusion and his number conservation tasks. More specifically, McGarrigle devised a number of experiments involving the class-inclusion tasks, but he differentiated the procedure, in that he changed the actual wording of the problems, so as to make them more meaningful and closer to the children's everyday experiences and interests. He found that even children aged between 3 to 5 years, were able to respond correctly to the tasks. Therefore, he argued, in Piaget' s experiments, children failed to deal with the class-inclusion task, because they had misinterpreted the problem and therefor, carried out the task differently from the way intended, and not because the lacked the conceptual ability to do otherwise. He concluded that the child' s interpretation of the task is of paramount importance in determining his/her performance (in: Huges, 1986).

This point was brought up in another study carried out by McGarrigle and Donaldson (1974). Although this time the study focused on Piaget's number-conservation tasks, the findings were somewhat similar to those evidenced above. More specifically, the researchers found that, when the rearrangement of the beads occurred 'accidentally' due to a 'naughty teddy', most 3 to 5 year-old children were able to conserve number and provide the correct response.

Both the above studies highlight the distinction that Donaldson made between the tasks requiring what she called 'embedded', versus 'disembedded' thinking. In particular, she argued that Piagetian tasks require that the child thinks about the language used by the adult, independently of the context in which it is being used (diembedded thinking), whereas, when children were asked to think within a context which made sense to them (embedded thinking), they were able to carry out the same tasks correctly (Grieve & Huges, 1990). The above issues have significant implications for understanding children' s difficulties with school arithmetic.

Even more direct evidence that young children have coherent number concepts, comes from the work of Gelman and her associates (Gelman & Gallistel, 1978 in Gomez, Piazza, Jobert, Dehaene-Lambertz, & Huron 2015). Much of Gelman's evidence comes from studies using the, so called, 'magic game' or the 'winner and losers' game. This game was initially designed as a test of the preschooler' s ability to reason about number. According to Gelman, the young child obtains representations of numerosity, by counting. In addition, the child' s ability to count is governed by a set of principles, which constitute a scheme, in that they both motivate the development of proficiency at counting. These principles are: (1) the 'one to one correspondence' principle, i.e. that the number words have to be assigned to each and every item to be counted, (2) the 'stable-order' principle, i.e. that number words have to be produced in the same stably ordered list, (3) the 'cardinality' principle, i.e. that the last number refers to the total number of all the items being counted, (4) the 'order-irrelevance' principle, i.e. that no matter which way we count a set of objects, the same number will be obtained and (5) the 'abstraction' principle, i.e. that we can count almost everything (Montague-Smith, Cotton, Hansen & Price, 2017).

Based on the above conceptualizations through her experiments, Gelman sought to determine if young children treated addition and subtraction as number-irrelevant transformations. In addition, Gelman differentiated her work from other similar studies, in that she tried to prove children' s understanding of number by looking at their types of errors, by talking to them about their ways of thinking when faced with several tasks, as well as by using interesting contexts and objects, such as puppets, whereby children were asked to 'correct; the puppet' s counting errors and to make predictions and guesses (Gelman & Gallistel, 1978).

On the basis of her studies, Gelman claimed that children as young as three years are able to understand the 'invariance' of small number arrays, that is, the fact that displacing the objects in an array, does not affect the numerosity in the way adding and subtracting objects does, thus being in sharp contrast with Piaget' s idea of conservation. She also maintained that many 3 and 4 year-old children understand the concepts of addition and subtraction, although as Huges (1991) argues, this claim is based on indirect evidence. Finally, the fact that 3 and 4 year-old children were able to detect errors in the performance of others, suggests that, even pupils as young as this, are able to monitor performance and therefore, presume an understanding of principles, given that the task is carried out in a meaningful and interesting context.

These observations call into question Piaget' s notion that the recognition of numerosity depends upon the development of hierarchical categorization abilities and suggest that teachers should be focusing on what preschoolers 'can' do, given the appropriate context, rather than characterizing them only in terms of their 'deficiencies'.

Nevertheless, Huges (1989, 1991) argues, if these conclusions are correct, we need to think again about other reasons why young children may encounter difficulties with school arithmetic. In his book “Children and Number” (1986), Huges provides an overview of the several studies carried out around the area of numeracy difficulties, both in Britain, as well as in the USA. By pointing out some of the most important findings of those studies (including those carried out by himself), Huges attempts to reveal the 'real' nature of children' s difficulties with number. Apart from the already mentioned studies which indicate that young children' s mathematical abilities are most likely to be elicited when the task has a 'meaningful nature' for the child, Huges stresses that of equal interest are those studies which claim that children' s difficulties can be seen as a result of: (a) the 'formal code of arithmetic' and/or (b) the 'context-free statements' within this code.

A study carried out by Corran & Walkerdine (1981), revealed that a child' s difficulty to understand questions such as “what does one and two make?”, is due to his or her inability to understand the underlying concepts of a language or a 'formal code' which, whilst containing familiar words, their meanings have no real relevance to that ways children use them in their everyday conversations. The second point Huges makes about the formal code of arithmetic, is that statements within the code are 'context-free'. This, he believes, is a second possible source of much of children' s number difficulties. Such statements, according to Huges, make no reference to any particular objects or entities, rather, they can be used to represent or stand for a whole range of objects. As a result, he states, children need to be taught procedures that can help them 'translate' and create 'links' between those novel, formal language of arithmetic and their existing concepts about number.

A plethora of studies, carried out by several researchers, have also revealed that many children do not seem to realize that the arithmetical symbols which they use in their workbooks, can also be used to represent quantities of objects or operations on the same quantities (Brown & Burton, 1978, VanLehn, 1983, Brannin, 1982). As Huges (1991) suggests, this is mainly due to children's failure to make 'translations' between symbols, on the one hand, and concrete objects on the other.

Based on the above conceptualizations and findings, Huges (1986), argued that, even though evidence suggests that, even before they attend typical school instruction, children are capable of grasping the beginnings of arithmetical symbolism, they might fail to do so, if teachers are not careful enough to introduce those symbols in appropriate ways. This, he suggests, an be achieved by involving pupils in interesting and meaningful activities, which not only emphasize both modes of representation (i.e. formal/abstract and concrete), but which also clearly introduce the links between the two.

All the above views have significant implications, if we consider the difficulties which mathematics can pose for learners with Specific Language Impairment (S.L.I.). Grauberg (1985) and Hutt (1986), describe a variety of problems, which children with this type of difficulty experience, such as: reduced memory capacity, problems with symbol systems, poor comprehension of relational items, uncertainty over words with multiple meanings, and so forth. However, the possibility that numerical thought is essentially non-verbal, suggested by recent experimental findings, might carry serious educational implications. In particular, the findings generated by the studies carried out by Starkey, Spelke and Gelman (1990), which show that a pre-verbal system allows mental representations of numerical information during early infancy, as well as the Wynn' s (1992, 1994) research findings, which show that prelingual infants are capable of perceiving and processing numerical information, given that this information contains reduced verbal demands, have significant implications for teaching mathematics to SLI children (in: Donlan, 1993). More specifically, as Donlan (1993: 102) suggests: “If current theories are correct, the linguistic deficits should not inhibit the formation of fundamental numerical concepts, despite the vulnerability of later acquired mediated systems”.

In addition, the data gathered by Mann' s (1994) study on the early development of children' s literacy and numeracy skills, show that the emergent literacy of nursery school-age children, interacts with their emergent numeracy in several important ways, but in a very different manner from the one it does with adults. These findings also have important implications for the teaching methods, through which teachers should make children aware of the adult meanings of both reading and number, without imposing these meanings to them.

Finally, Huges and his colleagues; findings presented above, also suggest that educators should encourage children to use all their intuitive methods for dealing with symbolic understanding, including finger counting, imaginary objects, as well as writing down idiosyncratic marks on paper (Gomides et al., 2018). All these methods make 'human sense' to children, as these are the basis for subsequent understanding of formal methods and they can also be a very useful tool for the teaching of language-impaired children, who face difficulties with symbolic understanding (Grauberg, 1995).

The “Social-Constructivist” Perspective

All the above theoretical views, which have highlighted the evidence suggesting that there is a serious gap between everyday and school arithmetic (Huges, 1991; Munn, 1994) and, which emphasize the close link between language and number, closely correspond to the 'social-constructivist perspective' of viewing children' s learning.

As Bryant (1995) states, the approaches deriving from this psychological perspective, concentrate upon the 'transmission' of knowledge and the people who adopt such approaches acknowledge Vygotsky (1978) as their leader. Vygotsky emphasized the roles of social interaction and verbal communication in the learning of mathematics. A central tenet of his theory is the idea that all learning is essentially social in origin and that, in order to understand a child' s higher mental processes, one must examine their social origins: “Any function in the child' s cultural development appears twice, or on two planes. First, it appears on the social plane and then on the psychological plane” (The 'Genetic Law of Cultural Development', Vygotsky, 1978, pg. 54). Vygotsky was also the first to propose the idea of the “Zone of Proximal Development” (ZPD), whereby children eventually manage to do on their own, what, at first, they could do with the help of an adult. He also introduced the notion of the 'Cultural Tools', which according to Vygotsky, are inventions which increase and transform the human intellectual power (Bozkurt, 2017).

A number of cross-linguistic studies have indicated that the 'number system' is one of those cultural tools. Miller & Stigler (1987) in particular, who compared the counting abilities of 4.5 and 6-year-old Taiwanese and American children, found that the former did a great deal better than the latter. Similarly, Carraher & Schliemann (1990), who examined Taiwanese and British children in a shop task which involved money, found that the latter were experiencing difficulty in using the decade structure to solve mathematical problems at any rate, as far as money was concerned. These findings suggest that the linguistic advantage helps Chinese speaking children, not just to count more proficiently, but also to grasp the relations between different levels of the decade structure and to use these relations to solve simple problems. Thus, the number system becomes a cultural tool, far earlier for them, as opposed to the English speaking children. The findings also indicate that the nature of the cultural tool affects the way that children learn about it, and so does the context within which they learn about this tool (in: Bryant, 1995). Added to the above, studies of Brazilian 'street' children (Carraher et al., 1985) and of other 'poorly educated' populations (Saxe, 1988, 1991, Nunes et al., 1993), have revealed that, although these children were very capable of carrying out mental arithmetic calculations, quickly and accurately, when those were related to real, everyday and meaningful tasks, they, however, experienced great difficulties with exactly the same, but in a 'formal type', calculations (in: Anghilieri, 1995).

Finally, a study carried out by Hunter et al. (1993), which explored children's perception of mathematics as an academic exercise, versus their perception of everyday life mathematics, also revealed a serious gap between their performances in the two situations, characterized by the children' s inability to transfer their mathematical knowledge from one situation to the other.

All the above findings closely correspond to Vygotsky' s conceptualizations regarding the social nature of learning, as well as in terms of the crucial role of language in learning and have enormous implications for the teaching of children with learning difficulties. In other words, the above positions acknowledge that mathematical activity is a social as well as a cognitive phenomenon and that everyday mathematical practices can differ from one social, cultural group to another (Krause, 2019). Therefore, the school' s task is to make sure that the links between the mathematics being taught in school and the child' s own previous mathematical experiences are made explicit and established very early in the primary school (Bonti, 2013). Added to this, as Hunter et al. (1993) suggest, one way this can be achieved, is by setting as homework, the task of finding a real world use for each of the formal mathematical techniques.

As Daniels and Anghilieri (1995) argue, if following Vygotsky, individual learning is to be construed as the internalized results of social activity, mediated through sign systems, such as speech, then the teachers' role should change from that of a transmitter of pregiven understanding to that of a mediator between informal meanings and those which are taken to be socially valued at a particular time.

Moreover, studies reporting findings which suggest that blind (Urwin, 1982), hearing-impaired (Wood et al., 1984) and language-impaired (Daniels, 1990) children, due to the nature of their difficulties, acquire numeracy in qualitatively different ways, also call for a reconsideration of Vygotsky' s view, who argued that the nature of social interactions in which children engage, influences the nature of their cognitive development. Hence, Daniels (1990) argues, teachers need to invent ways that will help them gain insight of the range of the possible routes by which children come to understand, and invent tools of analyzing learning that will permit the discovery of the individual' s requirements for effective teaching within his or her ZPD (Nikolaevskaya, 2017).

In line with the above argument, is Saxe' s (1991) approach, which builds upon both Vygotsky' s and Piaget' s constructivist treatments. Saxe' s main argument is that children' s peer interactions play an important role in the development of their cognitive understanding. Applying this view to the area of mathematics learning, he suggests that we need to create appropriate analytic units to capture the complex relations between peer processes and cognitive development and to use this knowledge for teaching purposes.

Another interesting view, comes from those researchers who have argued that, in order to bridge the gap between formal and everyday mathematics, educationalists can adapt several approaches that have proved to be useful in understanding children' s difficulties in other academic areas, such as reading, in mathematics teaching. In particular, Anghilieri (1995) suggests that the 'emergent writing' approach, often used in the teaching of literacy, can be applied to the area of mathematics teaching and can be termed as the 'emergent mathematics' approach (Atkinson, 1992). Such an approach, whilst focusing upon children' s already established knowledge about number, might also incorporate the features that have been identified by several researchers, as being crucial for dealing with children' s numeracy difficulties. These would be: placing tasks in meaningful contexts, helping children understand the nature and purpose of mathematical symbols, encouraging children to develop and explore a variety of mental and written strategies and, requiring children to reflect upon mathematical processes.

Finally, another approach, which also enables teachers to focus on the underlying processes, rather than the products of learning, is the one suggested by Daniels (1988). Daniels claimed that by 'borrowing' Goodman' s (1969) 'miscue analysis' of reading, teachers can devise a similar approach to be used for the assessment of mathematics. Thus, a miscue analysis of mathematics would: use children' s mistakes as a means for understanding their ways of dealing with mathematical tasks, would investigate the child' s past learning through the use of interviews and would explore the amount and type of instruction a particular child requires in order to make progress.

The 'Information-Processing Perspective

The emphasis given on the underlying processes, rather than the products of learning, characterizing the above conceptualizations, leads to the conceptualizations, leads to the consideration of the information-processing models of cognitive processes and of the implications of those models for the learning and teaching of mathematics (Barbieri, Rodrigues, Dyson & Jordan, 2019).

The information-processing psychological model characterizes children as active processors of information, who generate predictions and hypotheses about their world and constantly test these predictions against experience. According to Bryant (1995), proponents of this approach are mainly interested in finding out what is happening when children engage in cognitive tasks, what kind of mistakes they make and why. Moreover, as Daniels and Anghilieri (1995) state, information-processing approaches introduce concepts of planning, executive control and selective attention, emphasize the learner' s active involvement in the step-by-step processing of information (Wittrock, 1974), raise issues of 'automaticity' and claim that the human information- processing system is limited by constraints of various memory and processor speeds.

As Anghilieri (1995) states, there are three main features of the information-processing system, each of which has very direct implications for introducing young children to the world of formal arithmetic. These are: (1) 'Inductive Reasoning' (i.e. inferring general rules or patterns from a range of particular cases) and 'Deductive Reasoning' (i.e. the opposite process of inferring specific cases from a general rule). As research suggests, humans seem to show a preference in the first way of making sense of their world, whilst showing great difficulties with deductive reasoning. This has clear and major implications in terms of the ways teachers introduce children to the formal rules and procedures of school mathematics, as very often these rarely fit together with what children already know. (2) Limited 'working memory capacity' (Passolunghi, Živković & Pellizzoni, 2019).

Research community suggests that humans can hold only about seven separate pieces of information in their short-term memory (Miles, 1956). And, as Anghilieri (1995) argues, three additional features of our information-processing system might also contribute, either positively or negatively, to this structural limitation. These relate to the development of: (a) 'selective attention' (i.e. the ability to focus only on the relevant information), (b) 'structured knowledge', which is further divided into the ability to associate separate pieces of information and remember them as one ('chunking') and the ability to make connections between different parts of our knowledge ('elaboration'). Both these features of our memories are dependent upon processes of symbolic representation. Again, based on this knowledge, the implications for teaching children school mathematics are, at first, that we ensure that children become confident users of mathematical symbols and secondly, that these symbols should be meaningful and integrated into their already established knowledge. (c) 'Processing strategies', which can be further divided to general and domain-specific strategies and which appear to emerge gradually, often as extensions, modifications or combinations of already existing strategies. Finally, (3) lack of 'Metacognitive awareness and control', that is, becoming aware of our own intellectual processes and more in control of them (Flavell, 1981), have also been considered as a possible source of children' s numeracy difficulties, in that their actual problems might not be so much related to their inability to carry out taught routines, but more to their inability of being aware of when these are appropriate.

Three approaches, which a number of researchers have found to be useful for fostering children' s meta-cognitive abilities involve: (a) an adult explicitly modelling and explaining a strategy to a child, (b) encouraging children to reflect upon and record their achievements and, (c) encouraging children to work collaboratively with peers when solving mathematical problems (James, 1985, Saxe, 1991, in: Anghilieri, 1995).

Finally, Gagne (1965), by acknowledging the complexity of mathematical learning, identified several sub-skills, which function as elements of more complex tasks. These include: signal learning, stimulus-response learning, chaining, verbal associations, multiple discriminations, concept and principle learning and problem-solving (Darmawan & Suparman, 2019). The implications of this view for mathematical instruction are that tasks should be broken down into their component parts and taught in a hierarchical sequence (Daniels & Anghilieri, 1995). However, as Dockrell and McShane (1993) argued, such a 'task analysis' should not be a single list of objectives, taught to all children, following exactly the same sequence, but should be used as a process for clarifying and understanding the individual pupils' learning skills and difficulties, which will then serve as a tool for meeting their needs (Zhang et al., 2020).

Conclusions

Psychologists' research on children' s mathematical development and difficulties, quite neatly fall into three separate branches. Each of them, asks its own distinctive questions, each has its own theory and each employs its own empirical paradigms. However, as Bryant (1995: 3) argues, although all these approaches have made a great deal of progress, particularly during the recent years, “the subject as a whole has suffered from a certain lack of connection between the three”. Since there is no compelling theoretical or practical reason for this separation, it seems that, in order to better understand how children learn mathematics or fail to do so, and in order to help those pupils who experience difficulties in arithmetic, psychologists and teachers need to bring together the three perspectives and try to make simultaneous use of their most useful aspects (Gilmore, Cragg & Simms, 2020).

Moreover, as Resnick, Bill and Lesgold (1992) suggest, virtually all psychologists of cognition, whether they come from an 'individual differences', a 'developmental' or an 'information-processing' perspective, share the view that it is essential to try to identify individuals' thinking and reasoning 'competencies', independently of their level of performance, an any particular occasion (in: Demetriou et al., 1992).

Finally, it seems that researchers coming from different psychological backgrounds, have identified a number of key principles concerning children's mathematical learning and difficulties, which have important implications for the teaching of children with special educational needs in the cognitive area of mathematics. These have been briefly summarized by Anghilieri (1995) as follows: Teachers should start with real problems, before introducing children to formal arithmetic and mathematical teaching should be embedded in a variety of meaningful contexts (Ardi, Ifdil, Suranata, Azhar, Daharnis, & Alizamar, 2019). In addition, educators should encourage children to represent their mathematical understandings both verbally and symbolically, beginning with symbols of their own devising, they should allow and encourage children to develop their own mathematical strategies and finally, they should involve children in various kinds of dialogue which encourage awareness of and reflection upon mathematical processes (Selter & Walter, 2020).

References

Alabdulaziz, M., & Higgins, S. (2017). Understanding technology use and constructivist strategies when addressing Saudi primary students' mathematics difficulties. *International Journal of Innovative Research in Science, Engineering and Technology, 6*(1), 105.

Ardi, Z., Rangka, I. B., Ifdil, I., Suranata, K., Azhar, Z., Daharnis, D., ... & Alizamar, A. (2019). Exploring the elementary students learning difficulties risks on mathematics based on students mathematic anxiety, mathematics self-efficacy and value beliefs using rasch measurement. In *Journal of Physics: Conference Series* (Vol. 1157, No. 3, p. 032095). IOP Publishing.

Anghilieri, J. (1995). *Children' s mathematical thinking in the primary years: Perspectives on children' s learning.* Cassell.

Atkinson, S. (1992). *Mathematics with reason: The emergent approach to primary maths*. Hodder & Stoughton.

Barbieri, C. A., Rodrigues, J., Dyson, N., & Jordan, N. C. (2019). Improving fraction understanding in sixth graders with mathematics difficulties: Effects of a number line approach combined with cognitive learning strategies. *Journal of Educational Psychology*.

Bartelet, D., Ansari, D., Vaessen, A., & Blomert, L. (2014). Cognitive subtypes of mathematics learning difficulties in primary education. *Research in Developmental Disabilities, 35*(3), 657-670.

Bonti, E. (2013). *Ειδικές Μαθησιακές Δυσκολίες: Μια εναλλακτική προσέγγιση για όλους.* [Specific Learning Difficulties: an alternative approach for all]. Thessaloniki: Methexis.

Bozkurt, G. (2017). Social Constructivism: Does It Succeed in Reconciling Individual Cognition with Social Teaching and Learning Practices in Mathematics? *Journal of Education and Practice*, *8*(3), 210-218.

Bryant, D. P. (2005). Commentary on early identification and intervention for students with mathematics difficulties. *Journal of Learning Disabilities*, 38, 340-345.

Bryant, P. (1992). Children and arithmetic. *Journal of Child Psychology and Psychiatry, Vol.36, No.1*, pp.3-32.

Daniels, H. (1988). Misunderstandings, miscues and maths, *British Journal of Special Education, Vol. 15, No. 1*, pp.11-13.

Daniels, H. (1990). Number and communication difficulty: A Vygotskian analysis, *Educational Studies, Vol.16, No.1*, pp.49-59.

Daniels, H. and Anghilieri, J. (1995). *Secondary mathematics and special educational needs*. Cassell.

Darmawan, E. W., & Suparman, S. (2019). Design of Mathematics Learning Media based on Discovery Learning to Improve Problem Solving Ability. *Indonesian Journal on Learning and Advanced Education (IJOLAE)*, *1*(2), 20-28.

Demetriou, A., Shayer, M. & Efklides, A. (1992). *Neo-Piagetian theories of cognitive development: Implications and applications for education*, Routledge.

Dockrell, J. & McShane, J. (1992). *Children' s learning difficulties: A cognitive approach* (chapter 5: Specific difficulties with number), Blackwell.

Donlan, C. (1993). Basic numeracy in children with specific language impairment, *Child Language Teaching and Therapy, Vol.9, No.1.*, pp. 95-105.

Dowker, A. (2017). Interventions for primary school children with difficulties in mathematics. In *Advances in child development and behavior* (Vol. 53, pp. 255-287). JAI.

Fisher, R. (1990). *Teaching children to think.* Stanley Thornes Ltd.

Geary, D. C. (2000). Mathematical disorders: An overview for educators. *Perspectives*, 26, 6-9.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, 37, 4-15.

Gelman, R. & Gallistel, C.R. (1978). *The child' s understanding of number,* Harvard University Press.

Gersten, R., Jordan, N., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities*, 38, 293-304.

Gilmore, C., Cragg, L., & Simms, V. (2020). What can cognitive psychology tell us about the challenges of learning mathematics (and what do we still not know)? *Profession*, *18*, 19.

Ginsburg, H. P. (2003). Mathematics learning disabilities: A view from developmental psychology. *Journal of Learning Disabilities*, 30, 20-33.

Gomez, A., Piazza, M., Jobert, A., Dehaene-Lambertz, G., Dehaene, S., & Huron, C. (2015). Mathematical difficulties in developmental coordination disorder: Symbolic and nonsymbolic number processing. *Research in Developmental Disabilities*, *43*, 167-178.

Gomez, A., Piazza, M., Jobert, A., Dehaene-Lambertz, G., & Huron, C. (2017). Numerical abilities of school-age children with Developmental Coordination Disorder (DCD): A behavioral and eye-tracking study. *Human Movement Science 55,* 315– 26

Gomides, M. R. D. A., Martins, G. A., Alves, I. S., Júlio-Costa, A., Jaeger, A., & Haase, V. G. (2018). Heterogeneity of math difficulties and its implications for interventions in multiplication skills. *Dementia & neuropsychologia*, *12*(3), 256-263.

Grauberg, E. (1995). Language and early mathematics – ten years on, *Child Language Teaching and Therapy, Vol.11, No.1.*, pp.34-39.

Grieve, R. & Huges, M. (1990). *Understanding children*, Blackwell.

Hallahan, D. P., Lloyd, J. W. Kauffman, J. M., Weiss, M. & Martinez, E. A. (2005). *Learning disabilities: Foundations, characteristics, and effective teaching*. Boston: Allyn and Bacon.

Huges, M. (1991). Chapter 11: What is difficult about learning arithmetic?, in: Light, P., Sheldon, S. and Woodhead, M. (Eds). *Child Development in Social Context (2): Learning to think* (1991), Open University.

Hunter, J., Turner, I., Russell, C., Trew, K. and Curry, C. (1993). Mathematics and the real world, *British Educational Research Journal, Vol. 19, No. 1*, pp. 17-27.

Individuals with Disabilities Education Improvement Act (2004). 20 USC 1400. Washington, DC: US Government.

Kakia, D., & Kougioumtzis (2016). Dyscalculia and Advisory Educational Intervention (2016). *Journal of Regional Socio-Economic Issues, 6(2),* 92-105.

Kamii, C.K. And Declark, G. (1985). *Young children reinvent arithmetic: Implications of Piaget' s theory*. Teachers' College Press.

Krause, C. M. (2019). What You See Is What You Get? Sign Language in the Mathematics Classroom. *Journal for Research in Mathematics Education*, *50*(1), 84-97.

Montague-Smith, A., Cotton, T., Hansen, A., & Price, A. J. (2017). Mathematics *in early years education*. Routledge.

Munn, P. (1994). The early development of literacy and numeracy skills, *European Early Childhood Education Research Journal, Vol.2, No.2*, 5-18.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Nikolaevskaya, I. (2017). Multidimensional model of the ZPD as tool of analysis of the child's cognitive-personal dynamics of development while overcoming learning difficulties. *Revue internationale du CRIRES: Innover dans la tradition de Vygotsky*, *4*(1), 154-160.

Passolunghi, M. C., Živković, M., & Pellizzoni, S. (2019). Mathematics anxiety and working memory. *IC Mammarella, S. Caviola, & A. Dowker, Mathematics anxiety: What is known, and what is still missing*, 103-125.

Piaget, J. & Inhelder, B. (1996). *The psychology of the child.* Routledge and Kegan Paul, London.

Saxe, G.B., Geahart, M., note, M. & Paduano, P. (1991). Peer interaction and the development of mathematical understandings: A new framework for research and educational practice, in: Daniels, H. (1993). *Charting the agenda: Educational activity after Vygotsky,* Routledge.

Selter, C., & Walter, D. (2020). Supporting Mathematical Learning Processes by Means of Mathematics Conferences and Mathematics Language Tools. In *International Reflections on the Netherlands Didactics of Mathematics* (pp. 229-254). Springer, Cham.

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.

Wadsworth, B.J. (1989). *Piaget' s theory of cognitive and affective development, Fourth edition*. Longman.

Willis, S. (1990). *Being numerate: What counts?.*ACER.

Zhang, X., Räsänen, P., Koponen, T., Aunola, K., Lerkkanen, M. K., & Nurmi, J. E. (2020). Early cognitive precursors of children's mathematics learning disability and persistent low achievement: A 5-year longitudinal study. *Child development, 91*(1), 7-27.