Effect of Cognitively Guided Instruction on Primary School Teachers' Perceptions of Learners' *Conceptual Understanding of Mathematics*: Mombasa County, Kenya

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Abstract

The study investigated the effect of Cognitively Guided Instruction on primary school teachers' perceptions of their learners' conceptual understanding of Mathematics. The Cognitively Guided Instruction (CGI) is a professional development programme for teachers of mathematics, designed to focus on the way children develop concepts in Mathematics and how the teachers understand this and use it to structure instruction. The study used the ex post facto research design. Purposive sampling technique was used to select 16 teachers into the experimental group and the matched pair design to select 16 teachers into the control group. The Teacher Perception Questionnaire (TPQ) was used to gather the required data. In order to check for differences between the experimental and the control groups, a Mann- Whitney U test was used at a significance level of 0.05. The study established that the CGI had a significant effect on the teachers' perception of their learners' conceptual understanding of Mathematics.

Key words: Cognitively Guided Instruction, Perceptions of teachers, Conceptual understanding.

1.Introduction

Mathematics has maintained an enduring image as a field of knowledge that lends its resources to many intellectual pursuits and practical applications (Brown, Hodson & Smith, 2013). It plays a remarkable role in most fields of human endeavour, including Science, Mathematical and Technological activities as well as Commerce, Economics, Education and even Humanities. According to Okafor (2012), Mathematics is the science that enables scientists and technologists to derive relationships among different areas of knowledge such as Biological, Chemical, Geophysical and Physical Sciences. It is also handy to scientists as they seek to understand and explain natural phenomena (Okafor, 2012).

The degree to which the function and benefits of Mathematics are realised depends heavily on how learners are introduced and exposed to mathematical concepts and how they engage with the subject. According to Putman, Heaton, Prawat and Remillard, (2007) this can only be achieved if learners are engaged in exploring mathematical ideas through problem solving, working with complex situations in which they formulate and model problems, screen relevant from irrelevant information, organise information, make conjectures and test their validity. Research in Mathematics Education is preoccupied with exploring how to develop student teachers' understanding of Mathematics and convincing student teachers to teach Mathematics for understanding (Llewellyn, 2012). Arguably, the dynamic demands of the society require that learners be prepared to understand Mathematics while at the same time develop capacity to use and communicate Mathematics in their current and future lives (Sierpinski & Kilpatrick, 2012)

The Cognitively Guided Instruction (CGI) is an inquiry-based approach to teaching mathematics. It provides teachers with knowledge about the developmental stages of children's mathematical reasoning. This enables the teacher to plan mathematics instruction based on their learners' understanding and consequently guide them through greater mathematical reasoning and concept mastery. In a CGI classroom, learning is achieved by learners spending a considerable amount of time in problem solving and are not told how 'to do' mathematics. The latter entails providing learners with a formula, showing them how to apply and asking them to apply it whether they understand it or not. In a CGI classroom, the learners are taken through an open-ended experience whereby they mostly have more than one way of solving the problem before they report their solutions to peers and the teacher. This provides a perfect opportunity to correct misconceptions and structure instruction to enable the child to learn. In controlled studies conducted in the United States of America, CGI teachers taught problem solving significantly more and number facts significantly less, than control teachers did (Chambers & Lacampagne, 1994). Building on this, the researcher sought to establish the types of pedagogical practices the CGI teachers in Mombasa County were using to facilitate the development of conceptual understanding among the pupils. Due to the fact that the teacher uses the learners' mathematical thinking to facilitate learning, the teaching and learning activities used in class will depend on what is perceived by the teacher to be the learners' abilities.

Robinson-Cimpian, R, Ganley and Copur-Gencturk (2014) established that a relation exists between teacher perceptions and learners' achievement. This confirms earlier positions by Wayne and Youngs (2003) and Beswick (2013) that the achievement of school children depends substantially on the views the teachers they are assigned hold about their individual ability.

Research examining teacher quality confirms the logical conclusion that poor quality of pupils' learning correlates strongly with poor quality of teachers' delivery and their perceptions of the pupils. Ostensibly, teachers do not make use of the pupils' thinking frameworks in a way that meaningfully takes in new knowledge. This could be attributed to the teachers' background with regard to performance at teacher training level, poor knowledge of mathematics and negative attitudes towards the teaching and learning of the subject. The interventions designed to address the low achievement in the subject have an impact on the perceptions teachers hold of their learners' conceptual understanding of Mathematics and this affects their pedagogical practices. This study was therefore designed to determine the effect of the CGI on primary school teachers' perceptions of their learners' mathematical ability.

2. The Effect of Perceptions of Teachers on Teaching-Learning

DiYanni (2016) opines that perception reflects what one thinks they know and not just they think they 'see'. That perception is more complex than it typically appears as it involves interpretation and hence understanding. One of the dangers of perception is that sometimes things one sees appears different from the way they really are. Beswick (2013) suggests that mathematics teachers' beliefs about the nature of mathematics influences the ways in which they teach the subject and this has a direct impact on the learner's understanding. The distinction between teachers' views of the nature of mathematics and mathematician's views of the discipline is highlighted by Burton (2002) who argues that teachers, like mathematicians, may view mathematics as a discipline but regard the school subject differently. Consequently, teachers will choose the content to be taught and the way to teach it based on what they think is important for the sake of advancing through the education ladder. Thus there is a definite link between the teachers' perception of their learners' conceptual understanding of mathematics and the pedagogical practices the teachers employ during mathematics instruction.

According to Stipek, Givvin, Salmon and MacGyvers (2001) teachers who perceive mathematics as a tool for problem solving and are inquiry-oriented, take a dynamic view of the subject conceptualising it as a discipline that is continually undergoing change and revision. To such teachers, mathematics is a response to a problem solving activity and should therefore be taught in an environment where learners are engaged in activities that will facilitate the construction of mathematical concepts. Such activities require reasoning and creativity, gathering and applying information and the ability to communicate mathematical thinking and outcomes. Teachers with this kind of mental orientation understand that their role is to support and guide the process of constructing mathematical concepts. That they need to set contextual problem-based tasks that elicit thinking and have meaning to the learner.

There exists another school of thought that perceive mathematics as static body of knowledge involving a set of rules and procedures that are applied to yield one right answer. The learner is expected to perform procedures and manipulate symbols without necessarily understanding them. The teacher is in control. According to Stigler and Hiebert (1997), typically, a teacher reviews or introduces a procedure, demonstrates the use of the procedure through step-by-step instructions and finally assigns learners problems on which to practice the procedure. This approach is often driven by a desire to pass examinations while functionally having no relationship with the person and the context of the learner.

The designing of mathematics instruction is influenced by the way the teacher perceives the various aspects of mathematics, children's thinking and mathematics learning (DiYanni ,2016). Teachers who undergo a professional development course will still be affected by their own perceptions of their learners' abilities in mathematics. The current study recognises the value and potential of this aspect of teaching and learning in determining the instructional outcomes. Indeed according to Wayne and Youngs (2003), the interaction between the teacher perceptions of what mathematics is and what the learner is perceived to be capable of understanding can be an impediment to learning. According to DiYanni (2016), expectations often influence our perception. What and how we see things is based on what we expect and this can be misleading. In a teaching- learning situation, any perception that is off the reality can lead to permanent damage to the learner. Teachers in service need to go through experiences that will make them develop perceptions that build their capacity to help learners explore mathematics in a meaningful way. The Cognitively Guided Instructional approach offers rich experiences to the teachers and has a potential of affecting their perceptions of the conceptual understanding of Mathematics of their learners.

The Cognitively Guided Instruction (CGI), just like other cognitive learning theories, focuses on the need to understand cognitive development and use this to plan instruction. It is a professional development programme based on an integrated programme of research focused on: the development of learners' mathematical thinking; instruction that influences that development; teachers' knowledge and beliefs that influence their instructional practices; and the way that teachers' knowledge, beliefs, and practices are influenced by their understanding of learners' mathematical thinking (Carpenter et al., 1999).

The focus of the engagement is children's mathematical thinking. With this in mind, Cognitively Guided Instruction (CGI) is designed to create an enabling environment for teachers to continually think of their own knowledge of children's thinking. This results in the teachers forming their own frameworks of understanding and using the knowledge package presented by Carpenter, Fennema, Franke, Levi and Empson (1999). As a way of modelling inquiry, the teachers apply a 'learner-centred' approach, construct, and apply their understanding in their own unique ways. This open-ended approach, though a desired component of inquiry opens doors for various options when it comes to instruction. The quality of instruction and the extent to which it meets the desired objectives depends heavily on a teacher's level of conceptual understanding of mathematics, learners' mathematical thinking and the thinking presented by Carpenter et al. (1999). This is complex and has the potential of generating several possible outcomes with regard to the teaching and learning of mathematics. The teaching and learning activities developed and modelled by the teacher are based on the teacher's perceived understanding of the learner's ability and this may not necessarily be the actual ability of the learner.

3. Methodology

The study used the ex post facto research design. Specifically, the study used the criterion-group (also known as the causal-comparative) type of the ex post facto design. In this study, the criterion was the Cognitively Guided Instruction (CGI) training. The data was gathered using the Teacher Perception Questionnaire and analysed using both descriptive and inferential statistics. A sample of 32 subjects (16 subjects in the experimental group and 16 in the control group) were selected for the study by using the purposive sampling technique. A test at 0.05 significance level was done to the hypothesis:

> *There is no difference in perception of learners' mathematical ability between teachers who went through the CGI training and those who did not.*

The Mann-Whitney U test was used to check if there were statistically significant differences between the experimental and the control groups based on this hypothesis.

4. Research Findings and Discussions

4.1 Bio-data

The researcher found out that most of Early Childhood Education teachers in the geographical area of study were female. Indeed all the respondents for this study were female and this was not by design. Hence the bio-data analysis does not include gender consideration.

The distribution of teachers based on the classes taught were as in figure 1

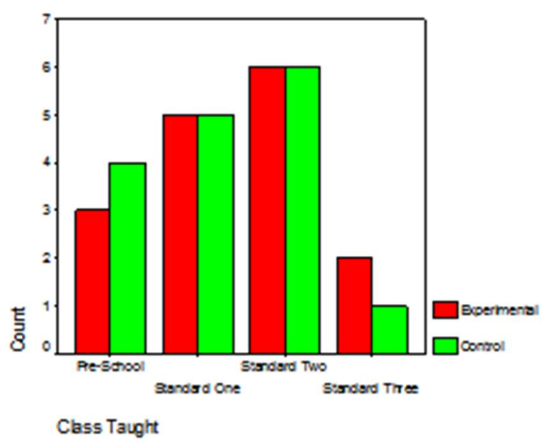

Figure 1: *Classes taught*

Professional qualifications was also an area of interest as it had the potential of contaminating the results. The results are in figure 2.

Professional Qualification

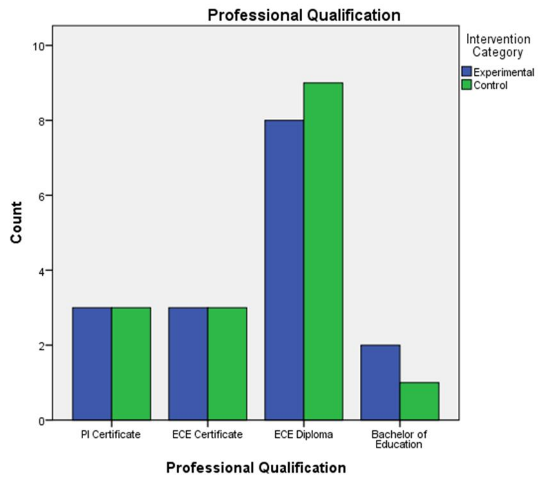

Figure 2: Professional Qualification

The results in figure 1 and figure 2 show that the members of the experimental and control groups were quite comparable, a clear result of the matched pair design that was used to obtain the control group. There is considerable similarity between the two groups in terms of classes taught and professional qualification. Combining the number of teachers holding Diploma or Bachelors degree yields equal numbers from both groups hence resolving the apparent difference between the ECE Diploma holders and the Bed graduates as seen in the graph.

This was also the situation with regard to number of years of teaching as displayed in Table 1

Table 1 Number of Years of Teaching

		Number of Years of Teaching			Total
		2-5 years	6-10 years	Over 10 years	
Intervention Category	Experimental	4	5	7	16
	Control	4	9	3	16
Total		8	14	10	32

The two groups were found to be suitable for the study as they were similar in all characteristics except the intervention (Cognitively Guided Instruction).

4.2 Teachers' perception of their learners' conceptual understanding of Mathematics

The teachers' perceptions of their learners' conceptual understanding was investigated using 10 items in the Teachers Perception Questionnaire (TPQ). These were divided into 3 categories namely Mathematical Representation, Mathematical Reasoning and Common Stereotypes. Common stereotypes was included here because they contribute to the development of perceptions and affect student learning outcomes (Tatar & Emmanuel, 2001).

4.2.1 Mathematical Representation

Mathematical representation covered mathematical signs, symbols, modelling using objects and use of diagrams to represent mathematical concepts.

The following items were used to capture the data:

i. Most learners are able to recognize different mathematical signs and symbols.
ii. The learners are able to interpret different mathematical signs and symbols correctly.
iii. Most learners are not able to model problem situations using objects.
iv. Most learners are able to draw relevant diagrams to represent concepts in mathematics.

The results of the analysis of these items are shown in Tables 2 and 3.

Table 2

Mathematical Representation

	Intervention Category	N	Mean Rank	Sum of Ranks
Most learners are able to recognize different mathematical signs and symbols	Experimental	16	21.22	339.50
	Control	16	11.78	188.50
	Total	32		
The learners are able to interpret different mathematical signs and symbols correctly	Experimental	16	23.44	375.00
	Control	16	9.56	153.00
	Total	32		
Most learners are not able to model problem situations using objects	Experimental	16	23.22	371.50
	Control	16	9.78	156.50
	Total	32		
Most learners are able to draw relevant diagrams to represent concepts in mathematics	Experimental	16	20.91	334.50
	Control	16	12.09	193.50
	Total	32		

From Table 2, it is clear that the mean rank of the experimental group was higher than the control group in for the four items. Testing for significance using the Mann Whitney U test yielded the results in Table 3.

Table 3

Result of Test of Significance on Mathematical Representation

	Most learners are able to recognize different mathematical signs and symbols	The learners are able to interpret correctly different mathematical signs and symbols	Most learners are not able to model problem situations using objects	Most learners are able to draw relevant diagrams to represent concepts in mathematics
Mann-Whitney U	52.500	17.000	20.500	57.500
Wilcoxon W	188.500	153.000	156.500	193.500
Z	-3.000	-4.333	-4.203	-2.802
Asymp. Sig. (2-tailed)	.003	.000	.000	.005
Exact Sig. [2*(1-tailed Sig.)]	.003[b]	.000[b]	.000[b]	.007[b]

The teachers from the Cognitively Guided Instruction group had relatively more positive perceptions of their learners' mathematical representation than those from the non-CGI group. Specifically this was in regard to recognizing different mathematical signs and symbols ($U=52.500$, $p=.003$); interpreting correctly mathematical signs and symbols ($U=17.000$, $p<.001$); modelling problem situations using objects ($U=20.500$, $p<.001$) and using diagrams to represent mathematics ($U=57.500$, $p=.005$). Brown (2008) argues that children rely more on their intuitive or experiential understanding of mathematics and lack the vocabulary and concepts needed to connect to school mathematics. They build their vocabulary as they engage each other in collaborative discussions and well planned lessons. This is probably the reason the CGI teachers were superior to teachers from the control group in how they perceived their learners with regard to these specific abilities.

4.2.2 Mathematical Reasoning

The second category of conceptual understanding of Mathematics with regard to conceptual understanding focused on mathematical reasoning. Table 4 shows a summary of the descriptive statistics on mathematical reasoning.

Table 4

Results on Mathematical Reasoning

Ranks				
	Intervention Category	N	Mean Rank	Sum of Ranks
Most learners demonstrate mathematical reasoning during instruction	Experimental	16	23.53	376.50
	Control	16	9.47	151.50
	Total	32		
The learners can generate examples of concepts with ease	Experimental	16	23.25	372.00
	Control	16	9.75	156.00
	Total	32		
Most learners generate non examples with ease	Experimental	16	21.59	345.50
	Control	16	11.41	182.50
	Total	32		
Most learners struggle to compare and contrast related concepts	Experimental	16	22.06	353.00
	Control	16	10.94	175.00
	Total	32		

Each of the four items under this category has a different mean ranks for the two groups. The biggest difference for these is in the first item (Most learners demonstrate mathematical reasoning during instruction), which records a mean of 12.53 for the Experimental group and 9.47 for the control group. The differences were tested for statistical significance and the results are shown in Table 5

Table 5

Results of Test of Significance on Mathematical Reasoning

	Most learners demonstrate mathematical reasoning during instruction	The learners can generate examples of concepts with ease	Most learners generate non examples with ease	Most learners struggle to compare and contrast related concepts
Mann-Whitney U	15.500	20.000	46.500	39.000
Wilcoxon W	151.500	156.000	182.500	175.000
Z	-4.398	-4.189	-3.254	-3.515
Asymp. Sig. (2-tailed)	.000	.000	.001	.000
Exact Sig. [2*(1-tailed Sig.)]	.000[b]	.000[b]	.001[b]	.000[b]

The teachers in the CGI group had relatively more positive perceptions of their learners abilities in mathematical reasoning compared with those in the non-CGI group. The differences between the two groups were statistically significant with respect to demonstrating mathematical reasoning during the lesson (U=15.500, p<.001); ability to generate examples with ease (U= 20.000, p<.01); ability to generate non-examples with ease (U=46.500, p=.001); ability to compare and contrast related concepts (U=39.000, p<.001).

Being conscious that children love to collect and sort things even before coming to the mathematics class, teachers can use strategies that will tap into this potential. It involves noticing, describing, comparing the attributes of things including people, objects and animals and they use colour, shape, texture, temperature and type among others (Brown, 2008). The CGI teachers having engaged the learners in using relevant manipulatives and having taken into consideration the knowledge and thinking of the learners would therefore be able to stretch them as they apply the learnt knowledge in generating examples and counter examples with relative success hence the difference in perception.

4.2.3 Common Stereotypes

The researcher investigated the following two items in this category and the results are in shown in tables 6 and 7:

i. The best pupils are those with organized and well written notes
ii. Most learners who are not organized are low achievers

Table 6
Results on Common Stereotypes

Ranks				
	Intervention Category	N	Mean Rank	Sum of Ranks
The best pupils are those with organized and well written notes	Experimental	16	22.00	352.00
	Control	16	11.00	176.00
	Total	32		
Most learners who are not organized are low achievers	Experimental	16	23.38	374.00
	Control	16	9.63	154.00
	Total	32		

The difference in the mean ranks for the two items are 11(22.00 and 11.00) and 13.75 (23.38 and 9.63). Table 7 contains the results of the test of significance of these differences.

Table 7

Results of Test of Significance on Common Stereotypes

	The best pupils are those with organized and well written notes	Most learners who are not organized are low achievers
Mann-Whitney U	40.000	18.000
Wilcoxon W	176.000	154.000
Z	-3.495	-4.296
Asymp. Sig. (2-tailed)	.000	.000
Exact Sig. [2*(1-tailed Sig.)]	.001[b]	.000[b]

The teachers involved in the study were a product of a rigid content-based system with clear 'one-size- fit- all' structures. Such experiences can lead to formation of stereo types and often affect the perceptions of most people. In the current study, the difference between the two groups was significant with regard to the two stereo types in Table 7. Jussim and Harber (2005) argue that self-fulfilling prophecy and erroneous teacher expectations lead students to perform at levels consistent with those expectations. Effort must be put in influencing teachers to develop positive expectations. The Cognitively Guided Instruction had a positive impact in this regard. Believing that learners who have well organised and well- written notes are better than the others may change the style of instruction and expectations by the teacher. The learners with poorly written notes risk suffering prejudice and this has a potential of affecting the learning environment.

5 Conclusion

From the findings of this study, it was concluded that the Cognitively Guided Instruction (CGI) had an effect on the teachers' perception of their learners' conceptual understanding of mathematics. This study established that, unlike the non-CGI teachers, the CGI teachers had more positive perceptions of their learners' abilities in conceptual understanding of mathematics.

REFERENCES

Beswick, K. (2013). Teachers' beliefs about school mathematics and Mathematicians' mathematics and their relationship to practice. Educational Studies in Mathematics, 79(1), 127-147.

Brown, T. (2008). Signifying “students”,“teachers” and “mathematics”: A reading of a special issue. *Educational studies in mathematics*, *69*(3), 249-263.

Brown, T., Hodson, E & Smith, K. (2013). TIMSS mathematics has changed real mathematics forever. For the Learning of Mathematics, 33(2), 38-43.

Burton, L. (2002).Recognising commonalities and reconciling differences in Mathematics Education. *Educational Studies in Mathematics ,50, 15 7-175*

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., & Empson, S.B. (1999). *Children’s mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Chambers, D. L & Lacampagne, L. (1994). Cognitively Guided Instruction.Teaching Children Mathematics. The National Council of Teachers of Mathematics. 1994. Retrieved January 04, 2016 from HighBeam Research: https://www.highbeam.com/doc/1G1-16352802.html

DiYanni, R. (2016). Critical and Creative Thinking: A Brief Guide for Teachers. West Sussex. John Wiley and Sons Ltd.

Jussim, L., & Harber, K. D. (2005). Teacher expectations and self-fulfilling prophecies: Knowns and unknowns, resolved and unresolved controversies. *Personality and social psychology review*, *9*(2), 131-155.

Llewellyn, A.. (2012). Unpacking understanding: the (re)search for the Holy Grail of Mathematics Education. Educational Studies in Mathematics, 81(3), 385–399. Retrieved from http://www.jstor.org/stable/41819603

Okafor, J. (2012, March). *Transforming The Nigerian Nation Through Science, Technology and Mathematics Education*. Presented at the Annual Conference of School of Sciences, Federal College of Education. Retrieved from http://works.bepress.com/jerome_okafor/2/download/

Putnam, R. T., Heaton, R. M., Prawat, R. S., & Remillard, J. (2007). Teaching mathematics for understanding: Discussing case studies of four fifth-grade teachers. The Elementary School Journal, 213-228.

Robinson-Robinson-Cimpian, J. P., Lubienski, S. T., Ganley, C. M., & Copur- Gencturk, Y. (2014). Teachers' perceptions of learners' mathematics proficiency may exacerbat e early gender gaps in achievement. Developmental psychology,50(4), 1262.

Sierpinski, A., & Kilpatrick, J. (Eds.). (2012). *Mathematics Education as a research domain: A search for identity: An ICMI study* (Vol. 4). Springer Science & Business Media.

Stigler, J., & Hiebert, J. (1997). Understanding and improving classroom mathematics instruction: An overview of the TIMSS video study. In ACER National Conference 1997 (pp. 52-65).

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. Teaching and teacher education, 17(2), 213-226.

Tatar, M., & Emmanuel, G. (2001). Teachers' perceptions of their students' gender roles. *The Journal of Educational Research*, *94*(4), 215-224.

Wayne, A. J., & Youngs, P. (2003). Teacher Characteristics and Student Achievement Gains: A Review. *Review of Educational Research*, *73*(1), 89–122. Retrieved from http://www.jstor.org/stable/3516044