THE INFLUENCE OF CONCRETE PICTORIAL ABSTRACT (CPA) APPROACH TO THE MATHEMATICAL REPRESENTATION ABILITY ACHIEVEMENT OF THE PRE-SERVICE TEACHERS AT ELEMENTARY SCHOOL

Hafiziani Eka Putri
Sekolah Pascasarjana Universitas Pendidikan Indonesia
Jl. Setiabudhi No. 229 Bandung Jawa Barat Indonesia 40154
Email: fatya_faiza@yahoo.com

Abstract

The aim of this research is to investigate the influence of CPA approach implementation in mathematical teaching to the mathematical representation ability (MRA) achievement of pre-service teachers (PT). This research was conducted using a quasi-experiment with pre-test and post-test control group. This research was also conducted using two study groups specifically study group with CPA as experimental group and conventional teaching and learning group as control group. The results show that the PT with CPA approach teaching and learning have MRA achievement significantly better than PT with conventional teaching and learning, if observed generally or observed through high and low of prior mathematical ability (PMA). The PT with intermediate PMA, there is no significantly different in MRA achievement of PT with CPA approach and PT with conventional teaching and learning.

Keywords: CPA approach, mathematical representation ability, pre-service teacher, elementary school.

1. INTRODUCTION

1.1 Background of Problems

One of the interesting topics that usually discussed by mathematical education expertise is the topic related to the mathematical ability of the PT or student in every education level. Mathematical ability is necessary to be developed due to its objective to improve the study performance quality and to stimulate the thinking model of student or PT in order to gain enough capability to face the new era challenges.

The mathematical ability is important for student has been stated explicitly on the objectives of The Curriculum of Level Education Unit in Indonesia (KTSP), namely that the students have the ability: 1) To understand the mathematical concept, to explain the correlation among the concepts, and apply the concepts or algorithms elegantly, accurately, efficiently and appropriate in the problem solving; 2) Using reasoning on model and character, to do mathematical manipulation to make generalization, to arrange evidence or explain idea and mathematical statement; 3) To solve the problem including problem understanding ability, creating mathematical model, solving the model and estimating the solution; 4) Communicating the idea with symbol, table of diagram, or other media to explain the situation or problem; 5) To have mathematical attitude in daily live, that is have curiosity, attention and interest in mathematical studying, also endurance attitude and self confidence in problem solving (Depdiknas, 2006).

The mathematical ability that must be mastered by student, of course it also must be mastered by PT that will teach mathematics. The *Committe in the Undergraduate Program in Mathematics* or CUPM (2004) gives six based recommendations for department, program and all

subjects in mathematics. One of the recommendations explains that every subject in mathematics should as activity that will help student in the development of analysis ability, critical logic, problem solving, communication and mathematical representative. Then, CUPM (2004) recommended that the duty of education institution should educate pre-service teacher who will teach mathematics having mathematics ability. NCTM (2003) explained that there are several mathematics ability supposed to be owned by student, one of them is mathematics representation ability.

However, some research show that majority of student failed to understand the necessity of correlation among several type of representation (Ainsworth in Hwang, et. al., 2007). This failure occurred due to the student seldom to be given an opportunity to explain a solution of problem both oral and written. Students only applying formula that has been studied to solve the problem, but not always understand the real concept or the basic principal behind the formula (Lesh, Baer and Forbes in Hwang, et. al., 2007).

Representation ability is necessary due to it has important role in student mind development, as the manifestation of educational process that have been undertaken by student. Rosengrant, et. al. (2007) stated that mathematics representation ability supports the student to achieve knowledge and problem solving. In line with this, Goldin (2002) expressed that construction of representation, representation system, and representational structure development are essential component for learning mathematics and problem solving. Therefore, the Institutions of Higher Education Teaching and Learning (LPTK) PGSD that served to educate pre-service elementary teacher who will teach mathematics is responsible to increase mathematics representative ability of the pre-service teacher.

To solve the lack of ability in mathematics representative of the student, NCTM (2000) suggested to student to be given opportunity to use several mathematics representative to solve problems with physical model, social and mathematics phenomena. One of the learning and teaching process which has characteristics that student has opportunity to use representation to solve the mathematical problem is CPA teaching and learning approach.

The CPA approach consist of three steps specifically: 1) learning by physical manipulation of concrete objects, 2) learning by representation of pictorial of concrete manipulation, and 3) solve the problem by mean an abstract notation (Witzell, 2005). Concrete components including manipulative objects such as cake, measurement tools, or the other object can be used during learning and teaching process. Pictorial representation is ability to create, read and graph or picture interpretation. The utilization of abstract notation is referring on symbolic representation such as number (numeral) or letters that written or interpreted when solve the problem (Sousa, 2007).

Sequence of learning activities carried out inthe CPA is very important. Activities with concrete material should take precedence to give the impression that mathematical operations can be used to solve real-world problems. Pictorial representation shows a visual representation of the manipulation of concrete help visualize mathematical operations in problem solving. This is important for the teacher to explain how the sample image relates to concrete objects. Then, a formal working with symbols are used to show how symbols providea shorter way and efficient to represent numerical operations. In the end, students need to achieve the highest level of abstract that is proficient in use of symbols with a lot of mathematical abilities they control.

Several researches support the CPA approach effectively. Witzell (2005) conducted a study of sixth and seventh grade students that are learning algebra. Students who learn how to solve algebraic equations transformation through the CPA approach obtain the test results higher than the control class (who received traditional instruction). The successful of CPA approach consistent for students with a history of learning achievement of low, medium, and high. In addition, students who

use the sequence of CPA approach in learning experienced procedural error in solving the problem less when solving algebra problems than traditional classroom variables (Witzel, Mercer & Miller, in Sousa, 2007).

Learning and teaching process using a CPA approach provides many opportunities for students to construct their knowledge. For example, interaction with concrete objects increases the possibility that students remember the choice of procedural stages in solving a mathematical problem, because it allows students to encode and retrieve informationin a variety of sensory options: visual, auditory, tactile (feeling/touch), and kinesthetic.

Based on the above explanation, therefore to test the reliability of the CPA approach in teaching and learning mathematics, hence it will be conducted a study that focused on developing a mathematical representation capability of elementary pre-service teachers through the CPA approach.

1.2 Formulation of the problem

Formulation of the problem in this study is described in the form of the following research questions: whether the achievement of mathematical representation ability of students who gain experience with the CPA approach is better than students who received conventional learning in terms of overall and Prior Mathematical Ability (PMA)?

1.3 Research Objectives

The general objective of this research was to determine the attainment of a mathematica lrepresentation of the ability of students who gain experience with the CPA approach is better than students who received conventional learning in terms of overall and Prior Mathematical Ability(PMA).

1.4 Research benefits

The benefits of this research are as follows:

1. Theoretically, this study will provide a reference enforce ability and reliability study of mathematics by CPA approach to the achievement of a mathematical representation of the student's ability pre-service elementary teachers.
2. In practice, this research will provide direct benefits to the faculty in developing the skills to teach mathematics to students of elementary teacher candidates. In addition, students as pre-service classroom teachers, gain direct experience of learning mathematics with application of CPA approach. This direct experience will probably be very useful to them when they practice in the real situation as teachers to design and implement a classroom teaching and learning. Another direct impact can be felt after the students learn mathematics with the implementation of the CPA approach, namely increasing the ability of the student mathematical representation it self.

2. Theoretical Studies

2.1 Mathematical Representation Ability

The concept of representation is one psychology concept that is often used in the field of mathematics education to explain some important phenomena of children's thinking. This concept is along tradition in philosophy which has been used with different connotations. For example as thought to reproducing the outside world or as a kind of view that an individual produces a mind of its own view point. The first representation is rooted in Cartesian philosophy and the idea of representation as are reflection of the soul, both of which are related to the idea of an individual

where the idea is derived by an individual from an object that is seen in the outside world (Radford, 1999).

In general psychology, representation means the modelling process of concrete things in the real world into an abstract concept or symbol. Furthermore, in the mathematical psychology, representation is defined as a description of the relationship between objects and symbols (Hwang, et al., 2007). Rosengrant, et al., (2007) suggested that the representation is a process to modelling or symbolizes an object (thing). Model or symbol can be made in the form of words, images, graphics, computer simulation, and mathematical equations and miscellaneos. By using a variety of representations, students can make connections, compare, develop and advance their understanding of mathematical concepts. Representation as physical objects, pictures, diagrams, graphs, and symbols also help students communicate their ideas (NCTM, 2000).

From several definitions above, in general representation is a configuration that can represent any particular case in some other way (a configuration object that can describe something in some way). In mathematics, for example, we can create a Cartesian graph as representation to reflect algebraic equations by writing the solution set or an equation can be illustrated by the graph in a way connects the coordinates.

Vergnaud (Goldin, 2002) stated that the representation is an important element in the theory of learning and teaching mathematics, not only because of the use of symbolic systems which are very important in mathematics, syntax and semantic rich, varied, and universal, but also to two strong epistemological reasons, namely: 1) mathematics plays an important part in making the concept of the real world, and 2) mathematics makes extensive use homomorphism in which the reduction of structure to each other is very important. Furthermore, it is said that the representation is an approach that makes the connection between an abstract more real by creating various types of configurations that have the connectedness properties that represents (Kaput in Goldin, 2002).

Kalathil and Sherin (2000) examined the benefits of representation for student learning. From the research found three things: 1) The representation can be used to provide information to the teacher and the class about how a student is thinking about a problem or mathematical context; 2) The representation used to provide information about the patterns and trends of all students in solving mathematical problems that are given; and 3) The representation can be used as a tool or a instrument for teachers and students to explore ideas in solving mathematical problems in the classroom.

There are five representations that used in mathematics education including representation of real-world objects, concrete representations, symbolic representation of arithmetic, verbal representation (language) and a graphical representation or image. Among these five things, the last three levels of representation is more abstract and higher in math problem solving (Lesh, Post, and Behr in Hwang, 2007). Verbal representation capability (language) is the ability to translate the characters and relationships that observed in math problems into spoken language. Graphical or image representation capability is the ability to translate mathematical problems into picture or graphic. Symbolic representation of arithmetic ability is the ability to translate mathematical problems in arithmetic formula(Hwang, et al, 2007). Furthermore, Cai, et al (Al Haddad, 2010) states that the variance representation which is often used in communicating mathematics are include: 1) visual presentation such as tables, pictures, graphs; (2) Statement of mathematics or mathematical notation; (3) a written text that written in both formal and informal writing.

From several mentioned representations it is can be drawn a conclusion that basically a representation can be grouped into: 1) the ability to translate the verbal representation of characters and relationships that observed inmathematical problem into the language spoken or written; 2) the ability to translate the visual representation of mathematical problems in the form of tables, figures

and charts); and 3) the ability to translate the symbolic representation of mathproblems in arithmetic formula, translating mathematical statement/mathematical notation.

The ability of a mathematical representation according to the NCTM (2003) are as follows: 1) Usingrepresentationsto model and interpret physical phenomena, social, and mathematics; 2) Create and use representations to organize, record (noted), and communicate mathematical ideas; and 3) Selecting, implementing, and translate mathematical representations to solve problems. Indicators of mathematical representation capability that will be used in this study will be made in line with the indicators according to the NCTM.

2.2 CPA Approach

Concrete-Pictorial-Abstract (CPA) sometimes is so called as Concrete-Representational-Abstract (CRA) or Concrete-Semiconcrete-Abstract (CSA). The teaching approach was similar and was originally based on the idea of Jerome Bruner in 1960. CPA approach consists of three stages where students learn through physical manipulation of concrete objects, followed by learning through a pictorial representation of a concrete manipulation, and ends with solving problems using abstract notation (Witzell, 2005). In line with the opinion of Witzel, Cooper (2012) describes three stages of learning sequences using CPA approach, namely: the initial phase involves students physically interact with the manipulation of concrete objects. The second stage involves working with a concrete representation of the model, which is usually a pictorial like circles, dots, counting, or geometric figures. The third stage is abstract stage that symbolically modeled using the concept of numbers, variables, and other mathematical symbols.

Stages of learning by using CPA approach are also described by Flores (2010), namely: First, manipulative objects are used to introduce the conceptual understanding. Second is concrete level that the learning proces sis described as follows: The instructor demonstrates the process/mathematical skills with manipulative objects, the instruct or then guide students to participate in the use of manipulative objects, giving directions and clues, and the students in dependently using object manipulation to demonstrate the skills/processes. Instructional the representational level follows the same steps, bu tmanipulative objects replaced with a picture and/or painting. After the representational phase, most of the interventions involving the CPA order give students strategies that help them to remember the steps in the process of mathematics. It serves as a transition from the use of drawings or paintings with the use of only the numbers are abstract phase. During the final phase, the students use the numbers in solving mathematical tasks, and instruction focuses on fluency.

CPA approach gives benefit to the most students and has proven to be very effective to help students who have difficulty in learning mathematics, because the CPA approach is moving gradually from actual objects throug him age and then subsequently to the symbol (Jordan, Miller, &Mercer, in Sousa, 2007). The students often feel frustrated when the teacher presents amathematical problem only in abstract form. Teachers need to develop a concept for aset of mathematical content and provide instruction that allows students to process new learning in a way that ismore meaningful and efficient.

Several studies support the effectiveness of this approach. Witzel (2005) conducted a study of students in grades ix and seven-identified as having difficulties in learning algebra. Students who learn how to solve algebraic equations transformation through the CPA approach obtained the test results higher than the control class (receiving traditional instruction). In addition, students who use the CPA sequence of procedures performed few ererrors when solving algebra problems variables (Witzel, Mercer, &Millerin in Sousa, 2007).

According to Cooper (2012) in teaching and learning with CPA approach there is manipulative aspects which are said to be a source of benefits and pitfalls at the same time, the benefits of learning with the use of manipulative objects will enhance the students' dispositions and attitudes towards learning in the classroom. The trap when students prefer to regard the use of manipulative objects as play events to fill the spare time than to provide an opportunity to improve their understanding of mathematics.

It can be concluded that teaching and learning using CPA approach is an learning approach that takes into account the sequence of the three stages of learning as a hierarchical circuit that begins with the use of concrete objects, then make a pictorial representation of concrete objects, and in the end the students work in abstract notation. By passing these three stages of learning, the student expected to understand the mathematical concepts clearly and correctly, and feel the benefits immediately when learning mathematics.

3. Research Method

This research design using quasi-experimental with pretest and posttest control group. Ruseffendi(1998: 45) described the design of such research areas follows:

O X O
O O

Description :
O = Pretest and posttest of mathematical representation ability.
X = Teaching and learning using CPA approach.
The research was conducted using two group of study specifically study group with CPA approach as experiment group and conventional teaching and learning as control group.

Before teaching and learning takes place both groups were given tests Prior Mathematical Ability (PMA). PMA test can be used to observe the effect of treatment of MRA achievement. Students PMA grouping a tall levels (low, medium, and high) based on the classification proposed by Arikunto (2012) as follows.

Table 1. PMA Level Category

Category	Range of Value
High ability	$x \geq \bar{x} + sd$
Intermediate ability	$\bar{x} - sd \leq x < \bar{x} + sd$
Low ability	$x < \bar{x} - sd$

(Description of the table: x= Score of student initial ability (PMA); $\bar{x}$ = Mean; dan sd = Deviation standard)

Population subjects in this study were all students of elementary teacher candidates in Universitas Pendidikan Indonesia (UPI) which consists of the central campus and regional campuses spread across two provinces namely West Java and Banten. The sample in this research are student pre-service teachers at the elementary level 2 semesters 4 which follows the mathematics education courses two at University of Indonesia Purwakarta campus. Sampling of class was done randomly from an existing class. Two classes used as a sample for the experimental group and the other two classes used as a sample for the control group. Subjects that are sampled in this study consisted of 69 students for the experimental group and 69 students for the control group.

4. Results and Discussion

Based on data analysis by descriptive and inferential, the average PMA students did not different between the groups of students who will get teaching and learning with CPA approach and

student groups that will get a conventional learning, viewed as a whole or as by groups of PMA. This situation is qualified to provide different treatment between the two groups of the study. If there is a difference in achievement of MRA after learning is done, the difference is due to the treatment (application of learning approach), it is not due to PMA. Testing differences of student MRA achievement can be conducted either as a whole, or a group of PMA, as the effect of the student PMA equality in both groups of teaching and learning (CPA and conventional).

4.1 Descriptive analysis of MRA data

The Achievement of Student MRA was seen from the average score of posttest (Suherman dan Kusumah, 1990). MRA student achievement criteria are grouped by using the combined criteria of Reference Benchmark Assessment (PAP) and Normative Assessment (PAN) (Suherman and Kusumah, 1990). The result of the calculation to determine the achievement MRA criteria presented in Table 2.

Tabel 2. The Criteria of Student MRA Achievement

Score Interval	Criteria
$\bar{x} \geq 32{,}69$	High
$17{,}45 < \bar{x} < 32{,}69$	Intermediate
$\bar{x} \leq 17{,}45$	Low

(Description: MRA ideal maximum score= 44)

The recapitulation of results analysis student MRA posttest scores based on the overall learning process is presented in Table 3.

Table 3. Recapitulation of Student MRA Posttest Score based on Learning Process

Posttest	Learning and Teaching	Score		$\bar{x}$	*sd*
		Lowest	Highest		
MRA (SMI=44)	CPA	12	44	29,985	7,517
	Conventional	9	42	26,304	7,912

(Description: SMI=Ideal MaximumScore)

Based on Table 3 it can be seen that the achievement of students MRA with CPA learning approach is higher than students who received conventional learning. Although the achievement qualificationof both study groups were on the same criteria, namely the intermediate criteria.

Table 4. The Recapitulation of Student MRA Posttest Score based on Learning Process Observed from PMA Group

Posttest	PMA Group	Teaching and Learning	Score		$\bar{x}$	*sd*
			Lowest	Highest		
MRA (SMI=44)	High	CPA	31	44	39,071	4,649
		Conventional	26	42	34,500	5,148
	Intermediate	CPA	12	44	27,574	6,299
		Conventional	10	42	25,541	7,526
	Low	CPA	20	40	28,250	6,364
		Conventional	9	37	22,182	6,853

Based on Table 4 it can be seen that student MRA achievement for every PMA Group with CPA teaching and learning approach is higher than the student with conventional teaching and learning. For each PMA group in the both teaching and learning group, the student MRA achievement for

high PMA group is on the high criteria, meanwhile for the intermediate and low PMA group are on the intermediate criteria. Table 4 also shows that student MRA achievement of high PMA group is higher than intermediate and low PMA group. Likewise the student MRA achievement who got the conventional teaching and learning on the intermediate PMA group is higher than low PMA group. However, contrarily the student MRA achievement who got CPA teaching and learning approach on the low PMA group is higner than intermediate PMA group.

Therefore, CPA teaching and learning approach has higher possibility to develop student MRA on every PMA group than conventional teaching and learning. Student MRA on high PMA group which got CPA teaching and learning more developed than the intermediate and low PMA group. Then, student MRA on low PMA group which got CPA teaching and learning approach is more developed than intermediate PMA group. The following is a graph presented clearly to express the differentiation of student MRA achievement based on teaching and learning group observed from PMA group.

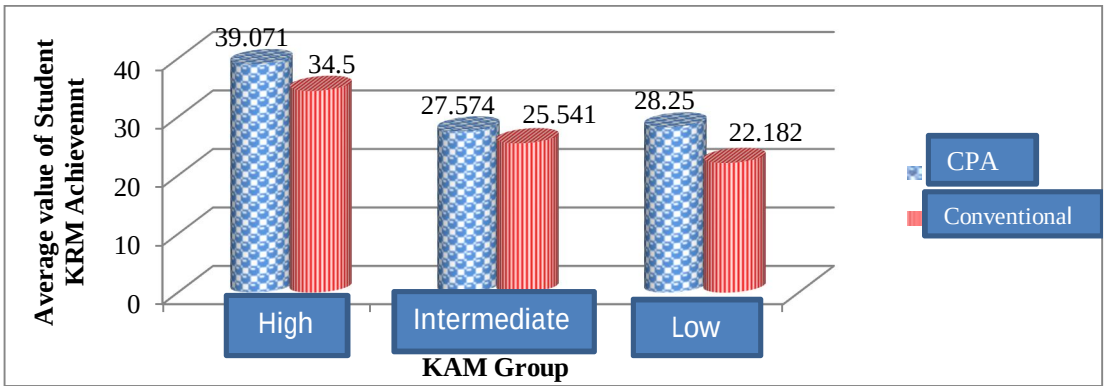

Figure 1. Student MRA achievement based on Teaching and Learning Observed from PMA Group

4.2 Analysis of MRA Data Inferential

It has been revealed descriptively that MRA achievement of student who got CPA teaching and learning is higher than student who got conventional teaching and learning either reviewed comprehensively or based on PMA groups. To examinewhetherthedifference of achievement is significantornot, then tested using the average difference test.

Before the differentiation test was conducted on student MRA achievement, the distribution normality test was conducted earlier on the achievement data in whole of both groups and reviewed from PMA group. Normality test was conducted using Kolmogorov-Smirnov (KS) method. If the data has normal distribution, then homogeneity test was conducted using Levene method. The hypothesis test criteria that used is if p-value (two-tailed) lower than $\propto = 0{,}05$, therefore the H_0 is rejected and in the other for the other H_0 is received.

If the data which will be tested on average difference of achievement and improvement has normal distribution and its variants is homogeny, then the different test that will be used is *t*-test. However, if the data has normal distribution but not homogeny, then the different test that will be used is *t'*-test. And then, if the data that will be tested one of them has abnormal distribution, then the average different test will be conducted using *Mann-Whitney* test.

Table 5. Normality Test of Student MRA Achievement Data based on Teaching and Learning

MRA Test	Teaching and Learning	*Kolmogorov-Smirnov*		
		Statistics	***df***	***p-value* (*two-tailed*)**
Achievement	CPA	0,122	69	0,012
	Conventional	0,069	69	0,200

From the Table 5 it can be seen that MRA achievement data with CPA approach has many sample from the abnormal distribution population. However, for MRA achievement data with conventional approach, the sample is resulted from normal distribution population. Due to one of the samples has MRA achievement data with abnormal distribution therefore the average difference test that will be used is *Mann-Whitney* test.

Table 6. Average Difference Test of Student Achievement based on Teaching and Learning on Whole Observation

Teaching and Learning	*Mann-WhitneyTest*	*Z*	*p-value* (*sig.*2-tailed)	Explanation
CPA	1777,500	-2,571	0,010	H_0 rejected
onventional				

Based on the calculation of Table 6, it can be seen that for MRA achievement average if observed as whole, student MRA achievement who receive teaching and learning using CPA approach is significantly better than student who receive conventional teaching and learning.

Table 7. Normality Test of Student MRA Achievement Data Observed from PMA Group

MRA Test	PMA Group	Teaching and Learning	*Kolmogorov-Smirnov*		
			Statistics	***df***	***p-value* (*sig.*2-tailed)**
Achievement	High	CPA	0,864	14	0,035
		Conventional	0,960	10	0,791
	Intermediate	CPA	0,979	47	0,537
		Conventional	0,988	48	0,906
	Low	CPA	0,267	8	0,098
		Conventional	0,230	11	0,107

.Based on Table 7 it is seemed that MRA achievement data from both of teaching and learning method for each PMA group commonly has normal distribution data, except on MRA achievement data observed from high PMA group for student who joined with CPA teaching and learning abnormal distribution data. Therefore, it is necessary to conduct the variant homogeneity test again for MRA achievement data on both teaching and learning method for each PMA group, except high PMA group, directly conducted non-parametric test, due to the data already as abnormal distribution. The non-parametric test that used is *Mann-Whitney test.*

Table 8. Recapitulation of Varian Homogeneity Test for Student MRA Improvement and Achievement Observed from PMA Group

MRA Test	PMA Group	Teaching and Learning	*n*	$F_{calculation}$	*p-value* (*sig.*2-tailed)
Achievement	Intermediate	CPA	47	2,504	0,117
		Conventional	48		
	Low	CPA	8	0,025	0,876
		Conventional	11		

Based on Table 8, it can be seen that for intermediate and low PMA group for student MRA achievement data has homogeny population varian. Then, it will be conducted an average different test with *t*-test for the whole of mentioned PMA groups.

Table 9. Recapitulation of Average Different Test Result of Student MRA Achievement Observed from High PMA Group

Teaching and Learning	*Mann-Whitney* Test	*Z*	*p-value* (*sig.*1-tailed)	Explanation
CPA	33,000	-2,182	0,015	H_0 rejected
Conventional				

Based on the data of Table 9, it can be seen that for high PMA group, student MRA achievement with CPA teaching and learning approach is significantly better than student with conventional teaching and learning.

Recapitulation of student achievement average different test result observed from low and intermediate PMA group can be seen on Table 10.

Table 10. Recapitulation of Student MRA Achievement Average Difference Test Result observed from PMA group

MRA Test	PMA Group	Teaching and Learning	Average	$t_{calculation}$	*df*	t_{table}	*p-value* (*sig.*1-tailed)	Explanation
Achievement	Intermediate	CPA	27,5746	1,426	93	1,886	0,078	H_0 accepted
		Conventional	25,5417					
	Low	CPA	28,2500	1,962	17	2,110	0,033	H_0 rejected
		Conventional	22,1818					

Based on the data from Table 10 it can be seen that for intermediate PMA group, student MRA achievement with CPA approach teaching and learning is not significantly different with student with conventional teaching and learning. For low PMA group the student MRA achievement with CPA teaching and learning approach is significantly better than student with conventional teaching and learning. Therefore, it can be concluded that student MRA achievement with CPA teaching and learning approach is significantly better than student with conventional teaching and learning if observed by whole and PMA group, except achievement for intermediate PMA group.

4.2 Discussion

Description or inferential analysis from the research result showed that achievement of student mathematics representation ability with CPA teaching and learning approach is better than student with conventional teaching and learning by observed through a whole or only from low and high PMA group. Meanwhile, inferential analysis for student with intermediate PMA showed that achievement of student mathematics representation ability who got CPA teaching and learning approach is not significantly different with student who got conventional teaching and learning. However, if observed descriptively student MRA achievement of intermediate PMA who got CPA teaching and learning is better than student who got conventional teaching and learning. Therefore, it can be said that the implementation of CPA teaching and learning approach influences to the student MRA achievement higher than conventional teaching and learning approach.

The highest MRA achievement is obtained by student in high PMA group who got CPA teaching and learning approach with high achievement criteria. Meanwhile, for student from low and intermediate PMA group who got CPA teaching and learning approach has achievement with intermediate criteria. The interesting phenomenon is that the CPA teaching and learning approach for student MRA achievement of low PMA group is better than intermediate PMA group. This condition can be happened due to the subsequence hierarchy of CPA teaching and learning approach, start from concrete manipulative object then student given an opportunity to make a pictorial representation from concrete object, and finally student work on abstract notation. By passing the three level of this teaching and learning, it is expected that student with low PMA will understands the mathematical concept clearly and accurately. This condition supported by opinion from Jordan, Miller, and Mercer (Sousa, 2007) who stated that CPA approach profitable to student and proofed very effective to help student who has difficulties in mathematical study since this CPA approach move gradually from real object then through figure and finally to symbol. Off course student with low PMA has higher difficulties in mathematical learning than student with intermediate and high PMA group. The student's learning difficulties mainly on MRA for low PMA group can be developed by implementing CPA approach in teaching and learning process.

5. Conclusion and Suggestion

5.1 Conclusion

The achievement of student's mathematical representation ability who got CPA teaching and learning approach is significantly better than student who got conventional teaching and learning approach observed both a whole or low and high PMA group.

5.2 Suggestion

The CPA teaching and learning approach will be better if used to develop student's mathematical representation ability mainly for student with low and high initial mathematical ability. Then, it is suggested to conduct research on the implementation of CPA approach in mathematics teaching and learning as an effort to improve others mathematical ability such as mathematical communication ability, mathematical connection ability, etc.

6. Reference

Alhadad, S. F. (2010). *Meningkatkan Kemampuan Representasi Multipel Matematis, Pemecahan Masalah Matematis, dan Self Esteem Siswa SMP Melalui Pembelajaran Dengan Pendekatan Open Ended*. Disertasi Doktor pada SPS UPI Bandung: Tidak diterbitkan.

Arikunto, S. (2012). *Prosedur Penelitian Suatu Pendekatan Praktek.* Jakarta: Rineka Cipta.

Committe in the Undergraduate Program in Mathematics (CUPM). (2004). *Six General Recommendations*.[Online]. Tersedia: http://www.maa.org/cupm/cupm2004.pdf [23 Maret 2012].

Cooper, E.T. (2012). Using Virtual Manipulatives with Pre-service Mathematics Teachers to Create Representational Models.*International Journal for Technology in Mathematics Education, Vol 19 No 3*. [Online]. Tersedia:https://ehis-ebscohost-com.ezp.lib.unimelb.edu.au/eds/pdfviewer/pdfviewer?vid=6&sid=cd03d495-1f99-4ec2-90d5-85ac8c67257b%40se ssionmgr115&hid=5.[20 Februari 2013]

Depdiknas. (2006). *Kurikulum Tingkat Satuan Pelajaran*. Jakarta: Depdiknas.

Flores, M.M. (2010). Using the Concrete–Representational–Abstract Sequence to Teach Subtraction With Regrouping to Students at Risk for Failure.*Journal : Remedial and Special Education Volume 31 Number 3 May/June 2010 195-207*. [Online]. Tersedia:http://resourcebinder802a.wikispaces .com/file/view/Effective+Math+Strategies+CRA.pdf.[20 Januari 2012]

Goldin, A. G. (2002). Representation in Mathematical Learning and Problem Solving. Dalam *Handbook of International Research in Mathematics Education editor Lyn D English*. New Jersey: Lawrence Erlbaum Associates, Inc

Hwang, dkk. (2007). *Multiple Representation Skills and Creativity Effects on Mathematical Problem Solving using a Multimedia Whiteboard System*. *Educational Technology & Society*, 10 (2), 191-212.[Online]. Tersedia:http://www.ifets.info/journals/10_2/17.pdf [2 Januari 2013].

Kalathil, R.R., & Sherin, M.G. (2000). Role of Students' Representations in theMathematics Classroom. In B. Dalam *Fishman & S. O'Connor-Divelbiss (Eds.),Fourth International Conference of the Learning Sciences* (pp. 27-28). Mahwah, NJ: Erlbaum. [Online]. Tersedia: http://www.umich.edu/~icls/proceedings /pdf/Kalathil.pdf [12 Desember 2012].

NCTM.(2003). *Program for Initial Preperation of Mathematics Specialists*. [Online]. Tersedia:http://www.ncate.org/LinkClick.aspx?fileticket=%2Frfx5Ju56RY%3D&tabid=676[12 Nopember 2012].

NCTM.(2000). *Using the NCTM 2000 Principles and* Standards *with The Learning from Assessmentmaterials*. [Online]. Tersedia: http://www.wested.org/lfa/ NCTM2000. PDF [2 Januari 2012].

Radford, L. (1999). Rethinking Representations. Dalam *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education*, North American Chapter, Universidad Autónoma del Estado de Morelos, México. [Online]. Tersedia: http://oldwebsite.laurentian.ca/educ/ lradford/RETHINKING.HTML[12 Desember 2012].

Rosengrant, D., dkk. (2007). *An Overview of Recent Research on Multiple Representations.* [Online]. Tersedia: http://paer.rutgers.edu/ScientificAbilities/Downloads/Papers/DavidRosperc2006.pdf[2 Januari 2013].

Ruseffendi, E.T. (1998b). *Dasar-Dasar Penelitian Pendidikan dan Bidang Non Eksakta Lainnya*. Semarang: IKIP Semarang Press.

Sousa, D.A. (2005). *The Concrete-Pictorial-Abstrak Approach*. [Online]. Tersedia: http://www.logan schools.org/mathframework/CPA.pdf [25 Nopember 2012]

Tall, D. (2002). "Advance Mathematical Thinking Processes", dalam *Advances Mathematical Thinking*(Editor Tall, D). USA: Kluwer Academic Publisher.

Witzel W.S. (2005).Using CRA to Teach Algebra to Students with Math Difficulties in Inclusive Settings.*A Contemporary Journal* 3(2), 49–60, 2005 .[Online]. Tersedia: https://ehis-ebscohost-com.ezp.lib.unimelb.edu.au/eds/pdfviewer/ pdfviewer?vid=7&sid=cd03d495-1f99-4ec2-90d5-85ac8c67257b%40sessionmgr115&hid=116[20Februari 2005]

CURRICULUM VITAE

Hafiziani Eka Putri received her Bachelor and Master degree in Mathematics Education from Universitas Pasundan and Universitas Pendidikan Indonesia (UPI), Bandung, Jawa Barat Indonesia in 2004 and 2006, respectively. She is currently as a PhD Candidate in Sekolah Pascasarjana Universitas Pendidikan Indonesia (UPI), besides that she is also a lecturer for Mathematics Education Undergraduate Program of UPI at Purwakarta. Her contact address is UPI campus Purwakarta Jl. Veteran Nomor 8, Purwakarta, Jawa Barat Indonesia. E-mail: fatya_faiza@yahoo.com.