

## **Development of Instructional Material Using Algebra as a Tool in Problem Solving**

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### **Abstract**

The main goal of mathematics is to make pupils effective problem solvers. Research-based grade level-appropriate instructional materials are therefore indispensable in order to develop the problem-solving skills of pupils in the elementary level. One area where students find greatest difficulty as revealed by Philippine National Achievement Test of Grade VI students is on fraction. The main objective of this study is to determine the effectiveness of a developed instructional material on fraction with the use of Algebra as a tool in problem-solving.

In achieving the objectives of the study, three instruments were developed. The instruments are the pre-test; the instructional materials and the post-test. The pre-experimental design was used where only one group was used to determine the effect of an instructional material. No control group was used since there is only one section to participate in the study.

Salient findings of this study are as follows: 1) the Grade V pupils exceeded the expected competency for Grade IV pupils to comprehend the meaning of fraction in spite of their poor performance in the pre-test; 2) the pupils employed varied strategies in solving problems such as use of algebra, division algorithm and drawing or modeling to find the part of a given number; and 3) the use of the instructional material using algebra as a tool in problem-solving increased the number of pupils using algebraic solution from 7.5% to 20.9% as revealed by the pre-test and the post test.

**Keywords:** Problem Solving, algebra, instructional material, elementary mathematics

### **1. Introduction**

Mathematics is an integral part of education and is a required subject in the curriculum across levels in the basic education level. In the elementary level, Mathematics requires more than just performing the four fundamental operations of addition, subtraction, multiplication and division, or using formula to find an answer to a given problem. One of the main goals is to make students effective problem solver.

In the past several decades, there had been significant advances in the understanding of the complex processes involved in problem-solving. Approaches like teaching for problem-solving, teaching through problem-solving and teaching about problem-solving have brought about significant improvements in the problem-solving skills of students. However, these improvements have not lifted the rank of the Philippines in both the National Achievement Test (NAT) and in international examination, vis-à-vis, the Third International Mathematics and Science Study (TIMSS).

The one-size-fits-all curriculum does not take into consideration individual differences of students. Understanding and teaching are two different things. Teachers need to understand what the learners need and find solutions to lessen barriers that contribute to the failure of the teaching-learning process. Mathematics as a subject is a form of communication and to teach Mathematics is to be able to communicate effectively. Teachers must be aware of the learning styles of their pupils, develop and deliver the lessons clearly and monitor if the delivery has been received as it was expected (Allsopp, et al, 2007).

One barrier to the attainment of the main goal of teaching Mathematics is the pupils' inability to solve word problems. In the study by Brizuela in 1999, she showed that difficulties encountered by second year high school

students were compounded when concepts are applied in Mathematical word problems (Brizuela, 1999). In the study of Velasquez in 2001, she found that majority of the students learn best through active problem-solving, asking questions and applying mathematics in personal and daily life (Velasquez, 2001).

Studies revealed that problems involving fraction is the most difficult for students (Cabrillas, 2010; Malasa, 2001; and Bañas, 2007). This was highlighted by the findings of Cabrillas in 2010 that even Collegiate level students have difficulty in solving problems on fraction. Rational expressions were the common difficulties of second year high school students according to Malasa, 2001. In the elementary level, the performance level of pupils in rational numbers was generally low (Bañas, 2007).

The above-mentioned researches underscored the need for the teachers to provide well-planned classroom activities with challenging problems that cater to the pupils' learning style (Velasquez, 2001). Varied teaching strategies aside from the usual lecture should be employed for sufficient internalization of the skills and to unblock pupils' difficulties in solving word problems in Mathematics (Brizuela, 1999; and Lina 2002).

Social cognitive theory suggests that self-efficacy is one basis of motivation (Bandura, 1977). Anyone who acknowledges his ability and has a high goal will perform better and have better motivation to learn than one who is unsure of his ability. One way to increase self-motivation is to make students feel successful in their daily activities. Hence the need for an instructional material composed of basic lessons on fraction to establish a strong foundation on the concept of fraction followed by lessons dealing with non-routines problems involving fraction. The use of Algebra in solving word problems will facilitate the students' quests for more effective strategy in solving problems. Once given the appropriate strategy, the students will feel confident and satisfied.

Algebra as a subject is offered to first year high school as basic subject in mathematics. This subject serves as the basic foundation in solving problems in other mathematics subjects like Geometry and Trigonometry. This subject makes use of symbols, usually letters of the alphabet, to represent numbers or quantities and express general relationships that hold for all members of a specified set. If Algebra is used as a tool in problem-solving in high school, it can also be made possible in the elementary. It is for this reason that the present researcher prepared an instructional material covering the most difficult area in mathematics coupled with the use of Algebra as a tool in problem-solving.

Early mathematicians suggest different types of problem-solving. Landa in his Algo-Heuristic Theory describes semi-algorithmic and semi-heuristic problems, processes and instructions. According to him, there are classes of problems for which it is necessary to execute operations in a well-structured, predefined sequence (algorithmic problems). For such problem classes, it is possible to formulate a set of precise unambiguous instructions (algorithms) as to what one should do mentally and / or physically in order to successfully solve any problem belonging to that class. There are also classes of problems (creative or heuristic problems) for which precise and unambiguous sets of instructions cannot be formulated. For such classes of problems, it is possible to formulate instructions that contain a certain degree of uncertainty (heuristics). Most textbooks used in elementary introduce problem-solving following Polya's steps such as identifying what is given, what is asked, the process to be used and performing the operation.

Given these scenarios in mathematics education, and the different approaches used by early mathematicians, there is still a great need therefore to equip our students with the necessary learning tools to improve and strengthen their mathematical competencies specifically problem-solving involving fraction. Instruction in the Mathematics classroom is a key contribution to students' achievement in Mathematics. However, if class period is insufficient, other mode of instruction and / or intervention can supplement. This is an attempt of the researcher to come up with an instructional material that will help in attaining the main goal of Mathematics, with emphasis on the use of Algebra as a tool problem-solving.

### **1.1 Statement of the Problem**

Recent developments in Mathematics Education have pointed out the necessity of additional instructional materials on problem-solving specifically those involving fraction. Using additional instructional material paired with an appropriate problem-solving strategy will not only minimize time, effort and money on the part of the teachers who want to provide enriched lessons and activities but also promote development and mastery of problem-solving skill. This will also develop confidence and a feeling of success on the part of the students.

This study therefore aimed to develop an instructional material on problem-solving involving fractions. Specifically, it sought answers to the following questions:

1. What is the performance of the subjects of the study in problem-solving in comparison to what is expected of the grade level?
2. What are the problem-solving heuristics employed by the subjects of the study?
3. What instructional material may be designed to help the subjects of the study improve their problem-solving performance in Mathematics?
4. What is the effectiveness of the instructional material developed in enhancing the performance of the subjects of the study in problem-solving involving fraction?
5. What assistance may be given to the subjects to improve their problem solving performance using Algebra as a tool?

## **1.2 Scope and Delimitation**

This study aimed to develop an instructional material in problem-solving involving fraction. The said instructional material used Algebra as a strategy in solving problems. The subjects of the study are the 23 Grade V pupils of the Philippine Normal University-Center for Teaching and Learning. The institution is situated in Taft Avenue, Manila. There is only one section per grade level in this University and there is only one Mathematics teacher for Grades IV to VI. The Grade V class was chosen because this is the Grade level that needs a strong foundation on problem-solving involving fraction. The Mathematics competency in fraction starts at this grade level. Once trained in Grade V, the students will be able to apply what they have learned when they go to Grade VI.

The instructional material used in the study includes teacher made lessons on fraction composed of two parts. Part I deals with a review of background knowledge on fraction needed for problem-solving and Part II is all about problem-solving. This part is composed of two lessons. Lesson I deals with routine problems and Lesson II deals with non-routine problems. Each lesson emphasizes the use of Algebra as a strategy in solving problems. Other topics in Grade V Mathematics are not included in the study.

Pre-test post-test results were analyzed to serve as a basis for determining learning gains after the intervention was introduced.

## **Gap Bridged by the Study**

Numerous researches were made on how to improve mathematics teaching. Schools spend a great deal of time teaching children the concept they know to do well in the operations of number but very little time teaching children how to solve word problems.

Researchers on performance of pupils in Mathematics reveal pupils' difficulty in problem-solving and therefore recommend the development of instructional material suited to their needs. Studies on performances of high school and college students showed a common difficulty on rational numbers. The instructional material herein proposed will fill the gap on problem-solving deficiencies as well as rational number difficulty.

Instruction in the Mathematics classroom is a key contribution to student achievement. However, if class period is insufficient, other made of instruction can supplement. Many studies on development of instructional materials were made using drill approach and modular approach but none of them focused on the use of Algebraic approach. The proposed instructional material on the use of Algebra as a tool focuses on fraction. This material will fill the gap between the pupils' difficulty and successful problem-solving.

## **1.3 Theoretical Framework**

The introduction of Algebra as a tool in problem-solving is based on the Adaptive Character of Thought (ACT) developed by John Anderson which focuses on memory processes. This theory made a distinction among three

types of memory structures: declarative, procedural and working memory. According to Adaptive Character of Thought (ACT), all knowledge begins as declarative information; procedural knowledge is learned by making inferences from already existing factual knowledge and working memory is that part of long term memory that is most highly activated.

The Adaptive Character of Thought (ACT) explains a wide variety of memory effects and account for higher order skill. It is guided by the principle that there is a need for identification of goal structure of the problem space before providing instruction in the context of problem-solving. It also provides immediate feedback on errors to minimize working memory load. Instruction with learning is adjusted to “grain size” to account for the knowledge compilation process and to enable the student to match the target skill with accurate execution.

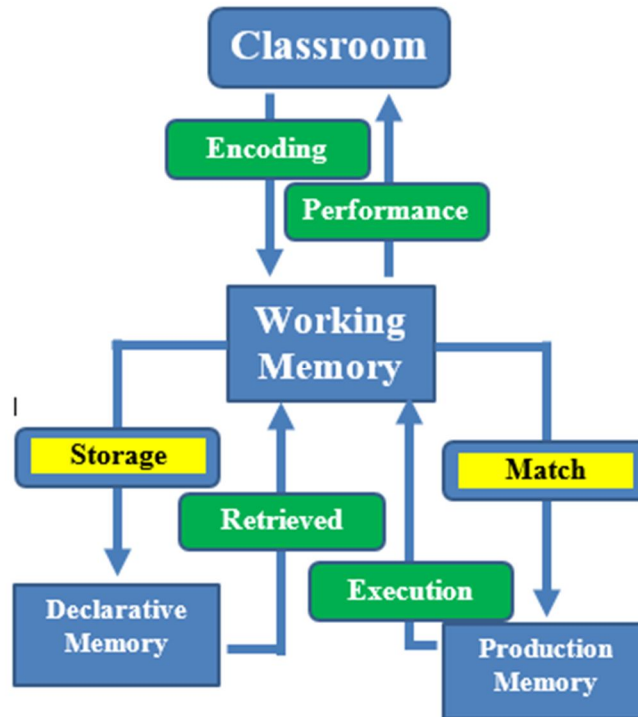


Figure 1. Theoretical Paradigm

#### 1.4 Conceptual Framework

This study considered the performance of Grade V pupils in problem-solving heuristics that they already know. This forms the basis of the development of an instructional material that focused on the use of Algebra as a tool in problem-solving. This instructional material is used involving Grade V pupils to determine its effectiveness in enhancing the problem-solving skill of the pupils. In order to better address the needs of the students, parts of the lesson that need improvement would be revised.



Figure 2. Conceptual Paradigm

The conceptual paradigm shows that the initial performance of the Grade V pupils and the heuristics that they already employ will serve as the basis for the development of instructional material using Algebra as a tool in problem-solving. Post-test will be conducted after the use of the material to determine the effectiveness of the material. Finally, the material will be enhanced based on the result of the study.

## 2. Methods

This study employed the pre-experimental design where only a single group was considered and no control group was included in the study. The single group was used to determine the effect of the use of Algebra as a tool in problem-solving. The group used as subjects of the study were the Grade V pupils in the Philippine Normal University-Center for Teacher and Learning. They are the grade V pupils during the S.Y. 2010-2011.

A teacher made pre-test was developed and validated by three content experts and language experts. They validated both the content of the test material, its grammatical construction and language appropriateness. The test was administered to the subjects to determine their initial performance level prior to the introduction of the instructional material. The instructional materials were prepared and are validated by experts as well. The same validation requirements as the test were required of the instructional materials: (1) content; (2) readability; and (3) language appropriateness.

The instructional materials prepared were all able to meet these requirements. After the validation of the instructional materials, these instruments were used for instruction in the pre-determined topics where students find greatest difficulty – fraction. After the instructional materials were completely administered, a post-test parallel to the pre-test was administered to the subjects of the study. The same content, readability and language appropriateness requirements were met by the post-test before administration to the subjects of the study.

### 2.1 Subjects of the Study

The subjects of the experiment were the 23 Grade V students presently enrolled at the Philippine Normal University-Center for Teaching and Learning for the school year 2010-2011. Of the 23 students, nine were boys and fourteen were girls with ages ranging from 9 ½ to 10 ½ years old.

### 2.2 Instruments

This study used three instruments. (1) the pre-test; (2) the post-test; and (3) the instructional materials.

(1) Pre-test and (2) post-test.

The researcher constructed a teacher made pre-test based on the table of specifications (TOS). The coverage of the pre-test is based on the competencies stated in the PELC for grade V Mathematics. Fifty items were constructed and submitted for critiquing.

Five Mathematics professors were chosen as jurors of the test, two from Bicol University and three from Philippine Normal University. Based on the comments and suggestions of the jurors on the face and content validity, the test was revised and was reduced to forty items.

The revised instrument was subjected to try out with the Grade V pupils of Bicol University College of Education Integrated Laboratory School after asking permission from the principal of the school.

After the try out, the papers were corrected and another revision was done. No changes were made on the number test items. They were only rearranged according to difficulty based on the result of the try out. Some problems were reconstructed to suit the comprehension of the Grade V pupils.

Table 1 shows the table of specification presenting the objectives / skills covered by the instrument.

**Table 1**  
*Table of Specification*

Objectives	Routine		Non-routine		Total Scores
	One-Step	Two-Step	One-Step	Two-Step	
1. To find fractional part of a number	1, 2	7, 8	11	12, 13, 14, 15, 16, 17, 18, 19, 20	<b>14</b>
2. To find what fractional part of a number is a given number	3, 4	9	21, 22, 23, 29	24, 25, 26, 27, 28, 30	<b>13</b>
3. To find the base given the fractional and the fractional name of the part of the base.	5, 6	10	31, 32, 36, 37, 39	33, 34, 35, 38, 40	<b>13</b>
<b>TOTAL</b>	<b>6</b>	<b>4</b>	<b>10</b>	<b>20</b>	<b>40</b>

Forty problems on fraction compose the pre-test. Of the 40 problems, 10 are routine problems and 30 are non-routine problems. Of the 10 routine problems six are one step and four are two step problems. While out of the 30 non routine problems, 10 are one step and 20 are two-step problems. After conducting the entire lesson a parallel post-test was given to determine the effectiveness of the use of Algebra as a tool in solving problems involving fraction. The same TOS was used to develop the post test.

### (3) Instructional Material on Fractions Using Algebra as a Tool for Problem-Solving

#### Features of the Instructional Materials

The instructional material emphasizes the use of Algebra as a tool in problem-solving. It is composed of three lessons.

**Lesson 1** deals with the development of background knowledge which concentrates on the skills identified in the Philippine Elementary Learning Competency (PELC). It includes definition of fraction, kinds of fraction, comparing, simplifying fractions and operations of fraction. This lesson is composed of seven sessions. In the operations of fraction, PELC does not included division of fraction. Since PELC contains only the required minimum learning competencies, this does not limit teachers to go beyond the minimum. It is for this reason that division is included in the development of the instructional material. Similarly, the first intent of this study was to develop instructional material for use by the Grade V pupils of the Philippine Normal University-Center for Teaching and Learning where the researcher is presently teaching. This class is used as training ground of pre-service teachers and therefore deserves enriched lessons.

**Lesson 2** deals with routine problems. It is composed of two sessions to discuss problems involving addition, subtraction, multiplication and division of fraction. Each session includes one- and two-step problems.

**Lesson 3** deals with non-routine problems. It is composed of three sessions to be discussed in six days. Three types of problems were discussed: Finding part of a given number, finding what fraction of the base is a given number and finally, finding the base. Table 2 shows the topics included in the study together with the time allotment.

**Table 2**  
*Topics in the Instructional Material with Suggested Time Allotment*

<b>Topics</b>		
<b>Lesson 1</b>	<b>Development of Background Knowledge</b>	<b>7 hours</b>
Session 1-1	Kinds of Fraction	
Session 1-2	Comparing Fractions	
Session 1-3	Simplifying Fractions	
Session 1-4	Addition of Fraction	
Session 1-5	Subtraction of Fraction	
Session 1-6	Multiplication of Fraction	
Session 1-7	Division of Fraction	
<b>Lesson 2</b>	<b>Routine Problems</b>	<b>4 hours</b>
Session 2-1	Addition and Subtraction Problems	
Session 2-3	Multiplication and Division Problems	
<b>Lesson 3</b>	<b>Non-routine Problems</b>	<b>6 hours</b>
Session 3-1	Finding part of a given number	
Session 3-2	Finding what fraction of the Base is a given number	
Session 3-3	Finding the Base	

### **Description of the Material**

This instructional material aims to develop the problem solving skills of Grade V pupils specifically in fraction which was found to be a very difficult topic not only for the elementary pupils but also for high school and college students. It introduces the use of Algebra as a tool in problem solving.

It is composed of three lessons. Lesson 1 concentrated on the development of background knowledge on the operations of fraction. This will prepare the students in problem solving and therefore needs to be mastered by the students. This will last from seven to eight days.

In Lesson 2 problem solving begins. It is composed of two sessions to be able to discuss problem solving involving the four operations. Problems include one-and two-step routine problems.

Lesson 3 deals with non-routine problems that can be discussed in six meetings at one hour each. It has three session plans covering finding part of a given number, finding what fraction of the base is the given number and finding the base. Each session included the problems to be presented for discussion, exercises for fixing skills, additional

problems for extension activities and assessment. The teacher may add more problems/exercises depending upon the abilities of the students.

### 2.3 Data Gathering Procedure

The researcher constructed a teacher made pre-test as described in the instruments section of this paper. The final copy of the teacher made pre-test was administered to 23 Grade V pupils of the Philippine Normal University–Center for Teaching and Learning, after having been given approval by the principal of the Philippine Normal University–Center for Teaching and Learning, the Dean of the College of Education and the Officer–in–Charge of the Office of the President of the Philippine Normal University.

Scores in the pre-test were determined using the rubrics as shown below:

**Table 3**

*Rubrics on Scoring Solutions to Word Problems*

Description	One-step Problem	Two-step Problem
No answer/ Wrong answer	0	0
Partially correct answer	1	1 – 2
Correct answer with solutions	2	3

After the pre-test, the instructional materials were developed. The three lessons were conducted in a span of 17 hours to teach the topic fraction. The first lesson includes seven sessions requiring an hour of delivery. The second lesson requires two sessions of two hours each and the last lesson requires three sessions with two hours each. At the end of the lessons, a post-test was conducted to determine learning gain.

### 2.4 Statistical Treatment of the Data

To interpret the results of the tests, mean and percentage were used. The mean was used to determine the performance level of the students in problem-solving involving fraction. The mean gain was also computed to determine the effectiveness of the use of Algebra as a tool in problem-solving.

The performance level of the students was interpreted based on the mastery level prescribed by the National Educational Testing and Research Center (NETRC) as shown below.

**Table 4**

*Mastery Percentage Level as described by the NETRC*

Competency Level (Range of Score in Percent Form)	Performance Level
92% and above	Excellent Performance
83% – 91%	Satisfactory Performance
75% – 82%	Average Performance
51% – 74%	Fair Performance
25% – 50%	Poor Performance
24% – and below	Very Poor Performance



## **4. Results and Discussion**

### **4.1 Performance of Grade V Pupils in Solving Problem Involving Fraction**

The Philippine Elementary Learning Competencies (PELC) serves as the framework of the curriculum for elementary in the country. As the name denotes, it spells out the competencies expected in every grade level. It serves as guide for the teachers who act as frontline instructional worker and school heads who act as instructional leaders in the delivery of instruction across disciplines and grade levels.

Based on PELC, the Grade V pupils are expected to demonstrate understanding of concepts and operations on fractions. These operations include addition, subtraction and multiplication of fraction. Discussion of each operation is followed by problem-solving as application. Division of fraction is not included in the coverage for Grade V.

These competencies expected from each Grade V pupil were discussed in this study as part of the lesson 1. Problem-solving on fraction which is given after each lesson on operation is concentrated on lessons 2 and 3 to cover routine and non-routine problems.

In an ordinary classroom setting, the lesson on operations of fraction is usually followed by word problems. After teaching addition of fraction, an application problem on addition of fraction follows. The same pattern holds true up to multiplication of fraction as stated in the PELC. However, some reference books include division of fraction. Most mathematics teacher especially those handling the highest section in central schools include division of fraction to prepare their students for future competitions. In this manner, the students immediately have a way to solve a given problem thus, making their task appear to be a routine exercise.

While it is true that problem-solving should be made a regular part of mathematics instruction to help pupils become critical thinkers and independent learners, teachers are expected to give non-routine problems. These kinds of problems do not give obvious solutions at first. Pupils should be trained to analyze the problem and look for information that may be helpful in determining the solution to the problem. It is in these processes that teacher introduce the use of Algebra as a tool in problem-solving.

The introduction of non-routine problems does not mean that routine problems will be excluded from the lesson. Routine problems will bridge the gap between operation of fraction and solving of non-routine problems.

The initial performance of Grade V pupils in problem-solving involving fraction was determined by a teacher made pre-test. Table 2 shows the score of each pupil in the teacher made test. From this table, it can be seen that the mean is 33.43 and the performance level is computed to be 32.14% which means poor performance level.

However, it is good to note that they still got a mean score of 33.43% despite the fact that this is their first exposure to problem-solving involving fraction. Based from the competencies for Grade IV as prescribed by the PELC, only comprehension of fraction is included. Operation of fraction is not part of what is expected of grade four students. Therefore, the performance of the pupils on problem-solving involving fractions exceeded the mathematics competency expected from Grade IV pupils.

For a detailed discussion of the performance level of the grade V pupils in problem-solving involving fraction, Table 3 is presented below. From the table it could be seen that five or 21.74% have very poor performance, 15 or 65.22% have poor performance while three or 13.04% have fair performance in problem-solving involving fraction. It is worthwhile to note that there are students who have advance knowledge in problem-solving involving fraction despite the fact that only concepts of fraction were discussed in Grade IV.

**Table 5**  
*Frequency of Pupils in each Performance Level in the Pre-Test*

Competency Level (Range of Score in Percent Form)	Frequency of Students	Percent	Performance Level
92% and above	0	0	Excellent Performance
83% – 91%	0	0	Satisfactory Performance
75% – 82%	0	0	Average Performance
51% – 74%	3	13.04	Fair Performance
25% – 50%	15	65.22	Poor Performance
24% – and below	5	21.74	Very Poor Performance
<b>Total</b>	<b>23</b>	<b>100</b>	

$N = 23$

Total Possible Score = 104

The findings on the very low performance of Grade V pupils in problem-solving involving fraction reveal a regional, if not a national issue regarding the dismal, if not depressing performance of students in Mathematics across levels. In particular, this confirms the findings of past researches that reveal about the difficulties of learners in various topics in mathematics across levels. Cabrillas (2010) found out that even college students have difficulty in solving problems on fraction. Bañas (2007) and Malasa (2001) found out the difficulties on rational number of learners in the elementary and high school levels, respectively. Lina (2002) found out that the Grade V pupils were very weak in analysis and organization of data in problem-solving. Bajado (1997) found out the serious misconception on the least common multiple (LCM) and greatest common factor (GCF) of integers. Fajardo (1997) also found out that third year student's are weak in Mathematical problem-solving particularly quadratic equation and rational expressions. Very few third year students were found to have knowledge of the different problem-solving strategies.

Although the pupils involved in this study had difficulties in problem-solving involving fraction, interventions like instructional materials using algebraic solutions are available to lessen the difficulties, thus help attain, if not exceed, the mastery level of the competencies expected in the grade level. With appropriate intervention, these difficulties in problem-solving need not be a persistent problem of teachers and school managers. These findings therefore make a call to the teachers with the support of the school heads to conduct classroom-based interventions utilizing strategies that address both the learning needs and styles of the pupils.

#### 4.2 Problem-Solving Heuristics of Grade V Pupils

Of the 40 problems given in the pre-test, 10 are routine problems and 30 are non-routine problems. Of the 10 routine problems, six are one-step problems and four are two-step problems. In solving these problems varied strategies were used by the students. Table 6 below summarizes the heuristics used by the Grade V pupils.

**Table 6**  
*Summary of Heuristics used by the Pupils*

Pupils	Algebraic Solution	Drawing / Illustration	Multiplication / Division	Answer Only
1	✓		✓	✓
2	✓		✓	✓
3	✓		✓	✓
4			✓	✓
5	✓			

6	✓		✓	✓
7				✓
8			✓	✓
9	✓			
10			✓	
11	✓		✓	
12	✓		✓	✓
13	✓		✓	✓
14	✓		✓	✓
15	✓		✓	✓
16	✓		✓	✓
17			✓	✓
18	✓		✓	✓
19				✓
20	✓	✓	✓	✓
21	✓		✓	✓
22	✓	✓	✓	✓
23			✓	✓
<b>Total:</b>	<b>23</b>	<b>16</b>	<b>2</b>	<b>19</b>

Sixteen pupils employed Algebra as a tool in problems which they have learned as early as June in their discussion of operation of whole numbers. Also, nineteen pupils used multiplication and division in finding part of a number while only two pupils used drawing in finding part of a given number. It can also be seen from the table that twelve pupils used two types of solution to solve the problem while two pupils used three types of solution. Of the 23 pupils, 19 were able to give correct answers to some problems without any solution.

#### **Problem-solving Heuristics Employed by Grade V Pupils**

The problem solving heuristics of the Grade V pupils shows that they can store knowledge in their long term memory. Their schemata in problem-solving in turn reveal the kind and scope of their exposure in mathematics instruction. It is therefore notable that the pupils were able to use algebraic solution in solving problems involving fraction considering their limited exposure to the use of algebra as a tool in problem solving.

In addition, these heuristics may imply the pupils' ability to comprehend the word problems. In other words, the pupils can translate the linguistic expression into mathematical expression, implying therefore that they know and understand what are given and what is asked. Whatever strategies they use in solving the problem, it is clear that they can figure out alternative ways to arriving at an answer. Indirectly, therefore, this implies that they have met the expected competencies in mathematics in the previous grade level.

Finally, the use of heuristics clearly indicates the pupils' attempt to solve the problems. Aside from considering the use of heuristics as a skill in itself, it is evident that the attempt to solve a problem using an alternative process, and not leaving a mathematical word problem unanswered, implies that the pupils possess positive attitude towards problem-solving in particular, and towards mathematics in general. This, in a way, suggests that the minimizing their difficulties may be facilitated because of their perseverance, resourcefulness and over-all positive attitude toward problem solving. Since the pupils possess the motivation to learn and master problem-solving, the teacher will not be any more confronted with a big problem in motivating the pupils to have practice exercises on routine and non-routine problems using Algebra as a tool.

#### **Effectiveness of Instructional Material in Improving the Grade V Pupils' Performance in Problem-solving Involving Fraction**

The effectiveness of the instructional material in fraction was based on the results of the pre-experiment conducted from 1<sup>st</sup> week of October 2010 to 3<sup>rd</sup> week of October 2010. The results of the teacher-made pre-test and parallel post-test were compared to determine the mean gain in the performance of the pupils. The mean gain served as the gauge in determining the effectiveness of the instructional material.

**Table 7***Performance of Grade V Pupils in Problem-solving Involving Fraction During the Pre- and Post-Test**n = 23*

Pupils	Score	
	Pre-Test	Post-Test
1	31	39
2	30	9
3	67	83
4	29	32
5	17	10
6	30	50
7	21	26
8	36	35
9	20	30
10	17	20
11	31	28
12	40	41
13	29	51
14	58	81
15	59	73
16	39	40
17	29	46
18	27	55
19	22	16
20	30	53
21	38	50
22	41	69
23	28	36
<b>Total</b>	<b>769</b>	<b>973</b>
<b>Mean</b>	<b>33.43</b>	<b>42.30</b>
<b>Performance Level</b>	<b>32.14%</b>	<b>40.67%</b>

*Standard Mastery Level: 75%**Mean Gain: 8.87**Total Possible Score: 104*

Table 7 shows the result of both the pre-test and the post-test to facilitate the determining of the mean gain. Based from the result of the post-test it was revealed that the mean increased from 33.34 in the pre-test to 42.30 in the post-test. Hence, there was a mean gain of 8.87.

**Table 8***Frequency of Pupils in each Performance Level in the Post-Test*

Competency Level (Range of Score in Percent Form)	Frequency of Students	Percent	Performance Level
92% and above	0	0	Excellent Performance
83% – 91%	0	0	Satisfactory Performance
75% – 82%	2	8.70	Average Performance
51% – 74%	4	17.39	Fair Performance
25% – 50%	13	56.52	Poor Performance
24% – and below	4	17.39	Very Poor Performance
<b>Total</b>	<b>23</b>	<b>100%</b>	

*N = 23**Total Possible Score = 104*

For more detailed interpretation of the pupils' level of performance in the post-test, Table 8 is presented. From the table, it could be seen that the very poor performers decreased from five or 21.74% to 4 or 17.39% in the pre-test to four in the post-test. The poor performers decreased from 15 or 65.22% to 13 or 56.52% and the fair performers increased from three or 13.04% to four or 17.39%. Two pupils or 8.7% have improved their performance from fair to average performance.

It was also noted from the result of the post-test that the number of pupils using Algebra as a tool in solving problem has increased from 7.5% during the pre-test to 20.9% in the post-test as reflected in Table 8.

Table 9 shows the number of students who used Algebraic solution, other solution and those who answered only taking only into consideration the partially correct and correct answer. In item number 1, for example, the table shows a total of 16 pupils with partially correct and correct answer. Of this number, 11 used Algebraic solutions; four used other solution while only one gave a correct answer without solution. In item number 16, a total of 12 pupils got partially correct and correct answer. Out of 12, 10 used Algebra as a tool in problem solving while only one presented other solution and one pupil also gave an answer without solution. This increase in the number of pupils using Algebra as a tool in problem solving suggests the effectiveness of using Algebra as a tool in problem solving to be able to arrive at the correct answer and to facilitate their solution to the problem.

**Table 9**  
*Frequency of Students who used Algebra as a tool in Problem Solving*

Items	Pre-Test			Post-Test		
	Algebraic Solution	Other Solution	Answer Only	Algebraic Solution	Other Solution	Answer Only
1	11	4	1	18	0	2
2	2	5	1	8	4	7
3	5	2	0	6	2	10
4	5	0	2	2	2	2
5	4	1	2	6	3	1
6	1	3	2	3	2	1
7	1	2	6	10	0	1
8	2	11	5	2	5	5
9	1	3	0	2	3	3
10	1	2	2	2	0	3
11	6	1	0	13	1	0
12	2	2	2	2	3	1
13	0	2	4	3	3	1
14	1	0	5	3	2	1
15	0	1	0	2	0	0
16	10	1	1	11	2	2
17	3	0	0	13	0	0
18	0	0	4	4	0	4
19	5	3	1	14	1	0
20	2	0	3	5	2	1
21	0	4	8	6	2	3
22	0	0	0	3	1	2
23	1	1	1	6	0	0
24	1	0	0	4	0	1
25	1	0	1	6	0	0
26	0	0	0	5	0	0
27	0	3	0	6	2	5
28	0	1	3	2	5	4
29	0	0	0	0	3	5
30	0	1	3	6	3	3
31	0	0	1	2	0	0
32	1	2	1	6	1	0
33	0	1	0	2	0	0
34	0	0	1	1	0	0

	35	0	0	1	2	0	0
	36	0	0	2	2	3	0
	37	0	1	0	1	1	0
	38	1	2	0	2	2	2
	39	1	0	0	1	3	2
	40	1	0	1	0	0	0
<b>Ave</b>	<b>920</b>	<b>7.5%</b>	<b>6.4%</b>	<b>7.0%</b>	<b>20.9%</b>	<b>6.6%</b>	<b>7.8%</b>

It can also be seen in Table 7 that there is an increase in the number of pupils using Algebra as a tool in problem solving in 38 out of 40 problems given in the pre-test and post-test. From this result, we can infer that the use of Algebra as a tool in problem solving has become part of the schema of the Grade V pupils. In other words, once a strategy has become part of the daily routine of the pupils, its use will become more spontaneous and effective as pupils progress to the next higher level of their schooling.

### Ways of Assisting the Pupils Using Algebra as a Tool in improving their Performance in Problem-Solving

The teacher plays pivotal roles in assisting pupils enhance their skills or minimize their difficulties. The same can be said in mathematics instruction. Cognizant of the merits of research-based grade level appropriate instructional materials together with time-tested strategies, the teachers can initiate classroom-based interventions which will aptly address the learning needs and styles of the pupils.

In elementary mathematics, the use of instructional material using Algebra as a tool may be considered as a simple yet enormous help in problem-solving. Considering the teacher-researcher's experience in the conduct of the experiment, it can be deduced that the proper use of the proposed instructional material as well as the continued use of Algebra as a tool in problem solving from the first unit to the last unit in Grade V Mathematics may improve the problem solving performance of pupils. In brief, it can be stated that Algebra served as a potent tool in improving the problem solving skills of the Grade V pupils.

Based on the experiment, the study was conducted after the lessons on Whole Numbers and Number Theory. At the start of the school year, the topic was operation of whole number. Right at the start, the teacher already used Algebra as a tool on problem solving. When the lesson on fraction began, there were already pupils who are used to using Algebraic solution without them knowing it. It was observed that constant practice on the use of Algebra as a tool in problem solving make their activities more enjoyable and effective. The same method was used up to the last topic in Math 5.

The use of Algebra as a tool in problem solving does not prohibit the students from using other strategies. In every problem, all strategies are accepted until finally the use of Algebra was introduced and made the pupils understand that it may be quite difficult at first. But, if they are already used to it, they will find problem solving routine and less frightening.

The use of Algebra as a tool in problem-solving is effective when introduced at the earliest possible time. In Grade V, Algebra as a tool should be used right at the start of the first unit which is whole number. And this should be used continuously up to the end of mathematics lesson of Grade V. When children are constantly exposed to this tool in problem-solving, they will find problem-solving satisfying and enjoyable. When the researcher started to introduce Algebra as a tool in solving problem in whole numbers, some children met difficulties but this was expected during the first exposure. This difficulty increased when parents interfered and resisted on the use of Algebra because they had difficulty in helping prepare the homework of their children. There were also negative comments from other teachers on the use of Algebra.

Despite these comments and resistance the researcher continued using it as a tool in problem-solving up to such time that most children already appreciated its use as a tool in problem-solving. This was proven by the result of the post-test where the number of pupils who use Algebra increase from 7.5% to 20.9%.

In mathematics there is nothing to memorize. What other pupils need is more practice to enhance their skill in computing. At this point, homework plays a very important role. What the pupils learn during the discussion is only stored in the short term memory. It is the homework that will help them become skillful and then knowledge will be transferred to the long term memory ready for retrieval as the need arises.

As revealed by the present study, using algebraic solution is an effective strategy of solving problems involving fractions. Previous researches also reveal other strategies found effective in lessening difficulties of learners in mathematics. Balbedina (2001) found out that the use of Practical Work Approach (PWA) as a teaching strategy was effective in developing problem-solving skill of third year students even if they are considered underachievers. Mapula (2001) found out that Damaths Game was effective in teaching integers to first year mathematics underachievers. Xin, Y. P., et al. (1999) found out that computer-assisted instruction yielded the largest effect size followed by representation technique and lastly, strategy training was found to be moderately effective for increasing student's problem-solving skills. Similarly, Mayfield and Chase (2002) argued that cumulative practice of component skills is an effective method of training problem solving.

## 5. Conclusions and Recommendations

### Conclusions

In the light of the foregoing findings, the following conclusions are deduced:

1. the result of the pre- and posttest showing very low performance of the grade V pupils in problem solving involving fraction can be explained by their first exposure to fraction, and limited exposure to the said Math topic in grade IV;
2. the pupils have varied problem-solving heuristics which were used as tools in solving word problems involving fraction;
3. it is further concluded that the instructional material included lessons and activities which require the pupils' active participation, exploration, and application of problem-solving skill in non-routine problems.
4. the lessons on fraction using Algebra as a tool in problem solving was effective in improving the problem solving performance of the pupils; and
5. Algebra served as a potent tool in improving the problem solving skills of the Grade V pupils. The teacher and the use of research-based grade level appropriate instructional material will play a crucial role in assisting the pupils improve their performance in problem-solving.

### Recommendations

Based from the findings and conclusions, the following recommendations are made:

1. The proposed instructional material should be used by Grade V pupils to enhance their problem-solving skills in fraction.
2. The giving of assignment or homework will help in developing the computation and problem solving skills of pupils. It is therefore recommended that the giving of assignment especially in Mathematics should not be prohibited.
3. The proposed instructional material can be extended to other units covered in Grade V Mathematics.
4. The material developed can likewise be used by grade VI pupils after class as a form of remedial lesson activity.
5. The material can be adopted for use by teachers and student teachers handling Mathematics V either as the main instructional material or as supplementary material on problem-solving.
6. Since the research-based instructional material was found effective, future studies on instructional materials in elementary mathematics using other approaches are strongly encouraged.

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